Equitable Handicapping in Golf

Derek R. Bingham and Tim B. Swartz *

Abstract

Previous studies on handicapping have suggested that in matches between 2 golfers, the better golfer has an advantage. In this paper, we consider medal play between 2 golfers when they are both playing well. The study uses the new slope system for handicapping. In this context we argue that it is actually the weaker golfer who has an advantage. The conclusions are based on both the analysis of actual golf scores and the analysis of theoretical models. We suggest an alternative scoring formula which leads to "fairer" competitions.

Keywords : data analysis, statistical modelling, order statistics.

*D. Bingham is Assistant Professor, Department of Statistics, University of Michigan, Ann Arbor MI, 48109. T. Swartz is Associate Professor, Department of Mathematics and Statistics, Simon Fraser University, Burnaby BC, Canada V5A1S6. T. Swartz was partially supported by a grant from the Natural Sciences and Engineering Research Council of Canada. The authors thank the Pemberton Golf and Country Club, Pemberton, British Columbia for access to their golf records. The authors also thank the editor and the associate editor for helpful comments that lead to an improvement in the manuscript.

1. INTRODUCTION

The purpose of handicapping in the game of golf is to allow golfers of varying skill levels to compete fairly. The fairness of handicap systems has been addressed by a number of authors. For instance, Tallis (1994) investigated handicapping in various team competitions and found that certain handicap systems can be extremely unfair. In this paper, we focus on matches between 2 golfers. In such matches, the general consensus is that handicap systems tend to favour the better golfer. For example, Scheid (1977) looked at 1000 scores obtained from members of the Plymouth Country Club in Plymouth, Massachusetts. Using simulated matches based on these scores, he calculated the winning proportion for the better golfer in medal play using handicaps. He found that with handicap differentials greater than 3, the winning proportion for the better golfer exceeded 60% and was as high as 85%. Pollock (1977) used a theoretical golf model based on the normal distribution and found that under reasonable variance assumptions, the better golfer has a competitive advantage in both medal play and match play. In a nice overview of the statistical literature pertaining to golf, Larkey (1998) explains that the pioneering work of Francis Scheid, beginning with Scheid (1971) was instrumental in initiating changes to the handicap system.

We address a slightly different problem in the context of medal play using the current handicap system. We are not concerned with the probability that golfer A defeats golfer B where golfer A is the better golfer. Rather, we are interested in the conditional probability that golfer A defeats golfer B when they are both playing well. Under a fair handicap system, the conditional probability that golfer A defeats golfer B should be .5. From a tournament perspective, this is a most practical issue. For a golfer does not expect to win a prize when he plays poorly, but when he plays well, at the very least, he expects a fair chance of winning a prize.

The central idea of this paper is that golfers with higher handicaps (i.e. weaker golfers) tend to have more variability in their golf scores. Consequently, the weaker golfer is more likely to post results well below (or above) their typical scores. Under the current handicap system (i.e. the slope system), this greater variability is not taken into consideration. We demonstrate that this gives an advantage to the weaker golfer when both golfers play well.

In Section 2, we briefly describe handicapping and indicate changes that have taken place in the handicap system since the papers of Scheid (1977) and Pollock (1977). Some real data are analyzed in Section 3. We see that in medal play using handicaps, the opposite effect to that observed in Scheid (1977) and Pollock (1977) takes place. That is, when both golfers are playing well, it is actually the weaker golfer who has an advantage in winning a match. The advantage is most dramatic when the better golfer happens to be a low-handicapper. In Section 4, we observe the same phenomenon by approaching the problem from a theoretical perspective. In Section 5, we describe a simple alternative scoring formula for use in tournament play and suggest that it is a more equitable method for prize distribution. We then conclude with some caveats and closing remarks.

2. HANDICAPPING IN GOLF

Since the publication of the articles by Scheid (1977) and Pollock (1977), there have been changes to the United States Golf Association (USGA) handicap system. Consider a golfer who has completed at least 20 rounds of golf and wishes to update his handicap. We refer to a gross score as a golfer's actual score obtained by summing his strokes over 18 holes. Using the old handicap system, a differential D_{Old}

is defined as

$$D_{\text{Old}} = X - R$$

where X is the adjusted gross score modified according to equitable stroke control (see Table 1) and R is the course rating for the course being played. Using the old handicap system, the golfer's handicap is calculated by averaging the 10 lowest differentials in the last 20 rounds, multiplying by .96 and then rounding to the nearest integer value. When playing on any course, a golfer's net score in medal play is then obtained by subtracting his handicap from his gross score. We note that only a golfer's lowest differentials (i.e. best rounds) are used in the handicap calculation. This is because the best rounds provide a better indication of ability; when a golfer is playing poorly, it is easy to lose concentration and have scores deteriorate. Using only the lowest differentials also provides a deterrent to the "sandbagger" who artificially attempts to inflate his handicap by intentionally playing poorly.

Since the early 1990's, the slope system has been gradually introduced to golf courses across North America as an enhancement to fair handicapping. The slope system generalizes the old handicap system by further considering course difficulty. Consider then a golfer who has completed at least 20 rounds where none of the resultant scores are deemed to be "eligible tournament" scores. The definition of the differential D is modified from $D_{\text{Old}} = X - R$ to

$$D = \frac{113(X - R)}{S}$$

where S is the slope rating for the course. A handicap index I is then obtained by averaging the 10 lowest differentials in the most recent 20 rounds, multiplying by .96 and truncating to 1 decimal place. When playing on a given course with slope rating S^*, the number of handicap strokes is now determined to be $I(S^*/113)$ rounded to the nearest integer value. If the competition is net medal play between 2 golfers, then the golfer's net score is obtained by subtracting the handicap strokes from his gross score. At this point we emphasize the distinction between the handicap index I and the number of handicap strokes for a given match; this distinction is important throughout the remainder of the paper.

In Table 2, we provide an example of the calculation of differentials under both handicap systems. Using the old system, the golfer receives $.96(158.3/10) \rightarrow 15$ handicap strokes independent of the course and the set of tees. Using the slope system, the golfer has a handicap index of $.96(154.8/10) \rightarrow 14.8$, and, for example, is allotted $14.8(120)/113 \rightarrow 16$ handicap strokes when playing from a set of tees on a course whose slope rating is 120.

Consider then 2 golfers who play most of their rounds on courses of average difficulty (i.e. $S \approx 113$) and then play an easy course (i.e. $S^* < 113$). From the definitions above, it is clear that they receive proportionally fewer handicap strokes under the slope system than under the old system. Similarly, they receive proportionally more handicap strokes under the slope system than under the old system when playing a difficult course (i.e. $S^* > 113$). If we accept the results of the existing literature that suggest that the better golfer always has an advantage, then it follows that the advantage on an easy course will be even more pronounced under the slope system.

For detailed information on the calculation of handicaps, eligible tournament scores, equitable stroke control, etc., see the USGA handicap web page at www.usga.org/handicap/manual. For information on the theory underlying the slope system, see Scheid (1995) and the references therein.

3. DATA ANALYSIS

Data were collected from the computer handicap system at the Pemberton Valley Golf and Country Club in Pemberton Valley, British Columbia during the 1997 golf season. Only full data on rounds played at Pemberton Valley were available. From the white tees, Pemberton Valley is a 5972 yard golf course with a course rating of 68.5 and a slope rating of 120. From the blue tees, Pemberton Valley measures 6407 yards with a course rating of 70.5 and a slope rating of 124. We demonstrate that in a competition between 2 golfers, the better golfer is disadvantaged in medal play when both golfers are playing well.

In this section, all reference to net scores should actually be interpreted as adjusted net scores where two adjustments are made. The first adjustment is due to equitable stroke control. Unfortunately, members enter adjusted gross scores into the computer database and there is no method of retrieving the (unadjusted) gross scores. However, we do not anticipate this to be a problem as we consider only the very best rounds of golf. Presumably, when a golfer is playing very well, the adjusted gross score will be equal or nearly equal to the gross score. The second adjustment is a standardization that transforms an adjusted gross score X_1 on a course with course rating R_1 and slope rating S_1 to an adjusted gross score $R_2 + S_2(X_1 - R_1)/S_1$ corresponding to a course with course rating R_2 and slope rating S_2. The standardization is based on the recognition that the differential $113(X_1 - R_1)/S_1$ is equivalent to the differential $113(X_2 - R_2)/S_2$. In our analysis, we transform scores from the blues tees to scores from the white tees at Pemberton Valley.

We limit our analysis to the 49 male members who completed 40 or more rounds during the year. We use the first 20 rounds as a tuneup period to allow the golfer to reach "mid-season" form and to allow the handicap index to settle. We also restrict our study to the immediate 20 rounds following the tuneup period. We hope that by using a shortened period, golfers will not experience dramatic changes in their skill levels. Each golfer will also have completed the same number of rounds of golf. Therefore our data analysis is based on $49(20) = 980$ scores.

For each of the 49 golfers, we calculate net scores for each of the 20 rounds subsequent to the initial 20 rounds of golf. We then take each golfer's best net score and use this as a measure of when the golfer is playing well. In Figure 1, we plot the minimum net score versus the handicap index I where the handicap index is calculated immediately after the tuneup period. We see that the plot has a negative regression line (slope = -.10) indicating that the weaker golfer has a better chance of defeating the stronger golfer when they both play well. A statistical test for the slope gives a marginally significant p-value of .049, where of course it is more difficult to establish an effect when the effect size is small. To put the plot into perspective, we would expect a 25-handicapper to beat a 5-handicapper by $(25-5)(.10) = 2$ net strokes in medal play when they are both playing their best games. This highlights the considerable inequity facing the low-handicapper in tournaments where only a few players win prizes.

Using the same data set, we again consider the 49 golfers and choose their best m net scores amongst the 20 rounds immediately following the initial tuneup period. Again, the net scores are based on the handicap index I determined at the end of the tuneup period. With m scores for each golfer, there are $\binom{49}{2} m^2$ possible matches between 2 golfers that can be simulated. The matches are simulated in the sense that the 2 golfers have not directly competed against one another. We consider $m = 2, 3, 4$ as this represents the best 10%, 15% and 20% of net scores (i.e. occasions when the golfers play well). We exclude from the analysis the 5 pairs of golfers that have the same handicap index. In Table 3, we give the results of the simulated matches, and again, we observe that the weaker golfer enjoys an advantage when both golfers are playing well. For example, with $m = 2$, the weaker golfer wins or ties 65.6% of the matches.

In Figure 2, we plot the sample standard deviation of the 20 net scores following the tuneup period versus the handicap index established at the end of the tuneup period. The least squares line has a slope of .07 and a corresponding p-value of .001. In this plot we see that the low handicap golfer is generally more consistent than the high handicap golfer. This result is not surprising. However, it does have implications for the fairness of matches when both golfers are playing well. This result, together with Figure 1, suggests that the very low handicap golfer has little chance of winning a tournament based on net scores.

We remark that these results are striking and contradict the USGA literature. From section 10-2 of the USGA Handicap Formula manual on the USGA web page, we quote, "As your Handicap Index improves (gets lower), you have a slightly better chance of placing high or winning a handicap event".

4. ANALYSIS OF A NORMAL MODEL

The use of the normal distribution in modelling golf scores has been previously considered in the literature. For example, Pollock (1977) used a normal model to investigate theoretical properties of handicapping. More notably, Scheid (1990) carried out an extensive analysis on the distribution of golf scores. Except for a slightly longer right tail, Scheid (1990) found the normal approximation to be satisfactory.

In this analysis, we define X_{ij} as the gross score corresponding to the i-th golfer on the j-th course with course rating R_j and slope rating S_j. We assume that scores are independent and that

$$D_{ij} = \frac{113(X_{ij} - R_j)}{S_j} \sim \text{Normal}[\theta_i, \tau_i^2]. \tag{1}$$

The normality assumption can be motivated by the Central Limit Theorem by noting that gross scores are obtained by summing the number of strokes taken over 18 holes. The assumption of the constancy of the distribution of D_{ij} over courses and tees is essentially the rationale for the slope system. Note that D_{ij} is not quite a differential since X_{ij} has not been modified according to equitable stroke control.

To test the adequacy of model (1), we calculate the Cramér-von Mises statistic W^2 and the corresponding p-value (page 122 of D'Agostino and Stephens, 1986) for each of the 49 golfers using the differentials obtained over the 20 post tuneup rounds. The results are consistent with the hypothesis of normality as only 2 of the 49 p-values (.046 and .047) are significant at level .05. When using the Anderson-Darling statistic A^2, only 1 p-value (.039) is found to be significant. With respect to the Scheid (1990) study, we observe that the slight departures from normality appear to be the result of a shorter than expected left tail. Although we recognize deficiencies in the model such as the approximation of a discrete distribution by a continuous distribution, the model provides insight on a number of handicap issues as well as motivation for a new scoring formula as developed in Section 5.

Now according to model (1), consider two golfers playing the same course so that we can drop the subscript j. Let H_i be the number of handicap strokes for the i-th golfer as described in Section 2, and without loss of generality let $H_1 < H_2$ such that the first golfer is the better player. Then

$$X_i \sim \text{Normal}[\mu_i, \sigma_i^2]$$

where $u_i = R + S\theta_i/113$ and $\sigma_i = S\tau_i/113$. Our interest lies in the investigation of

$$\begin{aligned} P_k &= \text{Prob(the better golfer wins} \mid \text{both golfers play well)} \\ &= \text{Prob}(X_1 - H_1 < X_2 - H_2 \mid X_i < \mu_i - k\sigma_i,\ \ i = 1, 2) \end{aligned}$$

where $k > 0$. Here, $X_i - H_i$ represents the net score of golfer i and we condition on both golfers playing better than k standard deviations below their average gross score. Since $\mu_i - H_i$ represents the average net score of golfer i, it follows from the results of Scheid (1977) and Pollock (1977) that

$$\Delta = (\mu_2 - H_2) - (\mu_1 - H_1) > 0.$$

When $\sigma_1 < \sigma_2$ and $k > \Delta/(\sigma_2 - \sigma_1)$, it is then easily shown that

$$P_k = \frac{\int_{z=-\infty}^{\frac{\Delta - k\sigma_2}{\sigma_1}} \phi(z) \left[\Phi(-k) - \Phi\left(\frac{\sigma_1 z - \Delta}{\sigma_2}\right)\right] \, dz}{\Phi^2(-k)}$$

where ϕ is the density and Φ is the cumulative distribution function of the standard normal. To get a sense of P_k for realistic values of k, consider typical values $\Delta = 1$, $\sigma_1 = 2$ and $\sigma_2 = 3$. Integrating via Simpson's rule, we obtain the decreasing probabilities $P_{1.0} = .39$, $P_{1.5} = .22$, $P_{2.0} = .10$ and $P_{2.5} = .03$.

Now let $G(z)$ be such that $G'(z) = \phi(z)\left[\Phi(-k) - \Phi\left(\frac{\sigma_1 z - \Delta}{\sigma_2}\right)\right]$ and $G(-\infty) = 0$. Then

$$\lim_{k\to\infty} P_k = \lim_{k\to\infty} \frac{G\left(\frac{\Delta - k\sigma_2}{\sigma_1}\right)}{\Phi^2(-k)} = 0$$

where the second equality follows from an application of l'Hospital's rule. Therefore, as both golfers play better (i.e. $k \to \infty$), it becomes impossible for the better golfer to win the match. This conclusion is in the same direction as the empirical results from Section 3.

Mosteller and Youtz (1992) considered the scores of professional golfers during the final 2 rounds of PGA tournaments under ideal weather conditions. They found that adjusted scores could be well approximated by a base score plus a Poisson variate. Whereas the Mosteller and Youtz (1992) analysis involved the most homogeneous of conditions, we are faced with data involving golfers of varying skill levels playing under various conditions. Furthermore, little is at stake for our golfers and we therefore do not expect their effort to be constant over all rounds. In an earlier version of this paper (which is available upon request), we extend the Mosteller and Youtz (1992) model such that the net score of the i-th golfer $X_i - H_i$ is given by

$$X_i - H_i = B_i + W_i$$

where B_i is the idealized or perfect base score for the i-th golfer and $W_i \sim \text{Poisson}(\lambda_i)$. This model leads to the same theoretical result that the better golfer is disadvantaged when two golfers play well.

5. AN ALTERNATIVE TO NET SCORES

In this section we propose a simple method for promoting fairness in matches. It is based on the normal model from Section 4 and consideration of the slope system. From model (1), it follows that

$$T \;=\; \frac{D_{ij} - \theta_i}{\tau_i}$$

has the same distribution (i.e. standard normal) for all golfers and ought to be a "fairer" yardstick than using the traditional net score. Moreover, T ought to work well generally (i.e. matches based on T should be fair unconditionally and matches based on T should be fair conditionally when golfers play well/average/poorly). Of course, T cannot be calculated since θ_i and τ_i are unknown. For simplicity of notation, we now drop the subscripts i and j. Our strategy then is to estimate θ and τ with $\hat{\theta}$ and $\hat{\tau}$ respectively, and then rate golfing performances using the quantity

$$T^* \;=\; \frac{D - \hat{\theta}}{\hat{\tau}} \; . \tag{2}$$

In obtaining $\hat{\tau}$, we refer to Figure 3 which plots the standard deviation of the differentials versus the handicap index I for each of the 49 golfers over the 20 post tuneup rounds. The handicap index I is established immediately after the tuneup period. Using a least squares fit in Figure 3, we have that $\hat{\tau} = \hat{\tau}(I)$ where

$$\hat{\tau}(I) \;=\; 2.74 + 0.053\, I. \tag{3}$$

Note that Figure 3 involves adjusted gross scores and therefore $\hat{\tau}(I)$ may slightly underestimate τ.

In obtaining $\hat{\theta}$, we recall from Section 2 that the handicap index I is calculated by

$$I = .96 \left(\frac{\sum_{i=1}^{10} D_{i:20}}{10} \right) \tag{4}$$

where $D_{i:20}$ is the i-th lowest differential in the last 20 rounds. We refer to $D_{i:20}$ as the i-th order statistic. Now a differential is only included in the calculation of the handicap index I when a golfer plays well. Presumably, under these conditions, a golfer's adjusted gross score equals his gross score where the adjustment is due to equitable stroke control. Therefore, using (4) and the normal model (1) without subscripts,

$$I = .096 \sum_{i=1}^{10} (\theta + \tau Z_{i:20})$$

where $Z_{i:20}$ is the i-th order statistic of the standard normal distribution. We therefore estimate θ by

$$\begin{aligned} \hat{\theta} = \hat{\theta}(I) &= \left[I/.096 - \hat{\tau}(I) \textstyle\sum_{i=1}^{10} \mathrm{E}(Z_{i:20}) \right] /10 \\ &= I/.96 + .7674\ \hat{\tau}(I) \end{aligned} \tag{5}$$

where the moments of the standard normal order statistics were obtained from the tables in Harter (1961).

Putting (2), (3) and (5) together, we propose the quantity

$$T^* = \frac{113(X - R)/S - 2.10 - 1.082\ I}{2.74 + 0.053\ I}$$

as a new scoring formula where small values indicate good rounds of golf.

We investigate the performance of T^* in the same way that we exposed the inadequacy of traditional net scores in Figure 1 and Table 3. For each of the 49 golfers, we calculate T^* for each of the 20 rounds subsequent to the initial 20 rounds of golf. Again, the handicap index I is calculated prior to each of the rounds of golf. We then take each golfer's best T^* and use this as a measure of when the golfer is playing well. In Figure 4, we plot the minimum T^* versus the handicap index. We see that the points are scattered in a horizontal band. A straight line regression gives an insignificant p-value of .27 for the slope. The lack of a pattern suggests that there are no handicap pairings for which a systematic advantage exists when using the new quantity T^*.

Using the 49 golfers, we also choose their m best T^* scores amongst the 20 rounds immediately following the initial tuneup period. Here the T^* scores are based on the handicap index I determined at the end of the tuneup period. We consider the $\binom{49}{2} m^2$ simulated matches between 2 golfers where $m = 2, 3, 4$ and exclude from the analysis the 5 pairs of golfers that have the same handicap index. In Table 4, we see that the outcomes of the simulated matches are far more balanced than when using traditional net scores. For example, with $m = 4$, the better golfer wins 50.3% of the matches. This is much closer to the idealized value 50% than the value $36.5\% + (1/2)13.1\% = 43.1\%$ obtained using traditional net scores.

Unfortunately, the average golfer may not not find the quantity T^* as intuitive and appealing as the traditional net score. However, in tournament play, we do not envision the individual golfer actively calculating T^*. This would be the responsibility of the prize committee. The situation is no different from tournaments where the prize committee privately selects random holes to establish individual handicaps and then determines net scores (e.g. the Peoria system).

We wish to stress that T^* is a first attempt at improving fairness in tournament play. The coefficients in T^* may be modified through a more comprehensive study of golf data. On the other hand, we have seen in Table 4 that T^* offers dramatic improvements over the traditional net scores and allows comparisons of scores from different courses. It is notable that the use of T^* does not require an overhaul of the handicap system; the ingredients in T^* are the course rating R, the slope rating S and the handicap index I, all of which are the same as before.

6. CONCLUDING REMARKS

We have demonstrated that a good golfer is disadvantaged in a tournament when prizes are distributed strictly according to net scores. It appears that this phenomenon has not gone unnoticed. As a result, there are typically various ways to win prizes in a large golf tournament. For example, the field of golfers is sometimes divided into flights so that only golfers of comparable abilities are competing against one another. Prizes are also often distributed according to gross scores within flights. Although these modifications are helpful, inequities are still bound to exist. For example, the first flight may group golfers in the 0 (scratch) to 8 handicap range where the 8 handicapper still has a distinct advantage over the scratch golfer when they both play well. Also, some tournaments are so small that it is not sensible to divide the field into flights.

We have focused on the case when golfers play well, and consequently, may win a tournament prize. In the less interesting case when both golfers play poorly, we expect the opposite results. That is, due to the greater variability in the scores of the weaker golfer, we expect that the better golfer is advantaged.

Also, we conjecture that the same general conclusions would be obtained in an analysis of match play using handicaps. To study match play, additional data is required. In particular, one needs the scores on individual holes.

In Section 5, we presented a simple scoring formula T^* that is more equitable than the calculation of traditional net scores. The basic idea underlying the approach is that golf scores become more variable as the handicap increases. However, there may be golfers who are consistent but are not very good. For example, certain senior golfers who hit the ball short distances but with high accuracy may fall into this class. For them, the proposed scoring formula may be penalizing. We suggest a more extensive collection of golf scores to further assess variability. Perhaps what is needed is a variability index V for every golfer. Like the handicap index I, V could be calculated from a golfer's best 10 differentials from their most recent 20 rounds. Following Section 5, a gross score X could then be transformed to a new net score $[113(X-R)/S - \hat{\theta}(I)]/V$. Of course, unlike T^*, such a proposal would require a complete overhaul of the USGA handicap system.

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Table 1: Equitable Stroke Control. The maximum score that can be reported on a hole for a golfer of a given handicap.

Handicap	Maximum Score
≤ 9	double bogey
10-19	7
20-29	8
30-39	9
≥ 40	10

Table 2: An example showing the calculation of differentials under the old handicap system (D_{Old}) and under the slope system (D). Here X is the adjusted gross score, R is the course rating and S is the slope rating. The asterisks indicate the differentials that are included in the handicap calculations.

	X	R	S	D_{Old}	D
	90	70.1	116	19.9	19.4
	91	70.1	116	20.9	20.4
	94	72.3	123	21.7	19.9
	88	70.1	116	17.9*	17.4*
	89	70.1	116	18.9	18.4
	90	72.3	123	17.7*	16.3*
	91	72.3	123	18.7*	17.2*
	91	70.1	116	20.9	20.4
	91	70.1	116	20.9	20.4
	86	68.7	105	17.3*	18.6
	90	70.1	116	19.9	19.4
	92	72.3	123	19.7	18.1*
	85	68.0	107	17.0*	18.0*
	78	68.7	105	9.3*	10.0*
	82	70.1	116	11.9*	11.6*
	84	70.1	116	13.9*	13.5*
	94	72.3	123	21.7	19.9
	93	72.3	123	20.7	19.0
	89	72.3	123	16.7*	15.3*
	88	70.1	116	17.9*	17.4*
Sum				158.3	154.8

Table 3: Simulated matches between 2 golfers based on their best m out of 20 net scores. The percentages refer to matches won, lost and tied by the lower handicap (i.e. better) golfer.

m	Matches	Wins	Losses	Ties
2	4,684	34.3%	52.8%	12.8%
3	10,539	35.4%	51.1%	13.5%
4	18,736	36.5%	50.4%	13.1%

Table 4: Simulated matches between 2 golfers based on their best m out of 20 scores using the statistic T^*. The percentages refer to matches won, lost and tied by the lower handicap (i.e. better) golfer.

m	Matches	Wins	Losses	Ties
2	4,684	47.9%	52.1%	0.0%
3	10,539	49.0%	50.9%	0.0%
4	18,736	50.3%	49.7%	0.0%

Figure 1: Minimum Net Score Versus Handicap Index

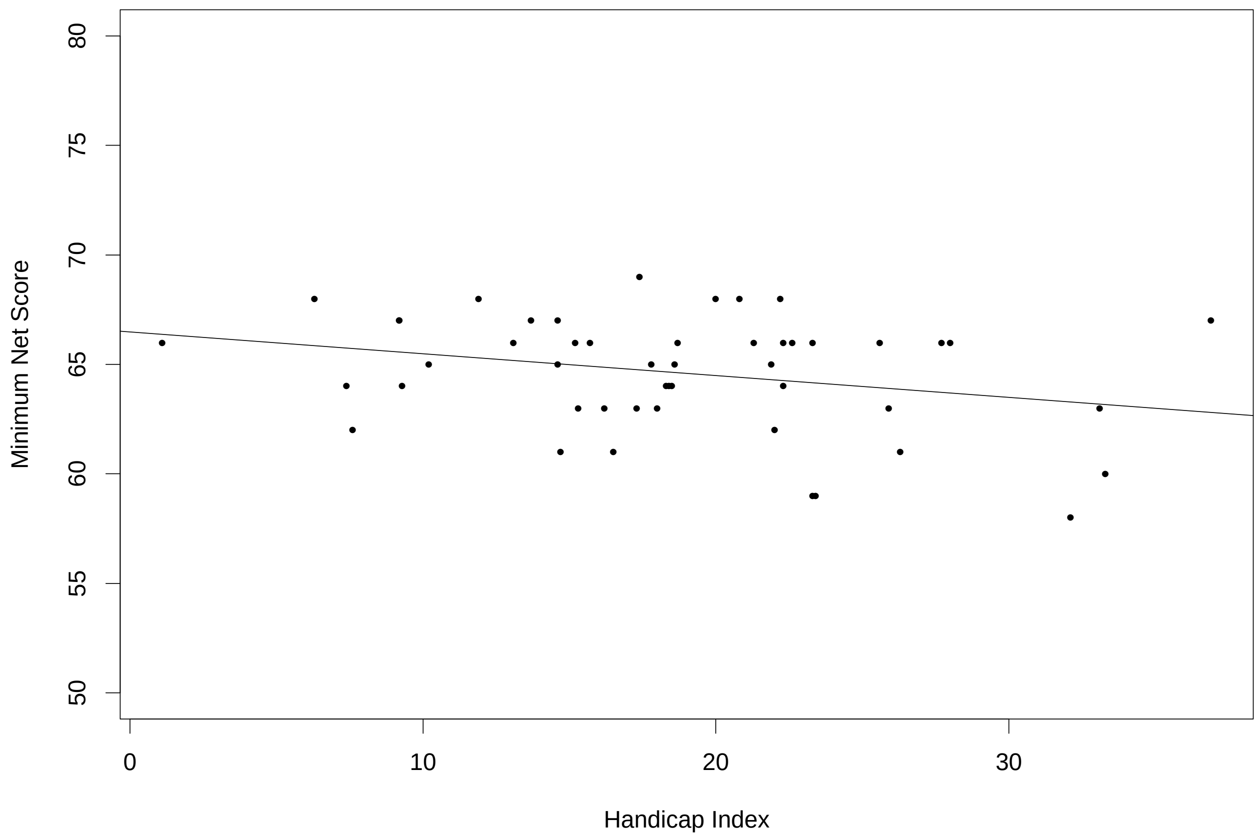

Figure 2: Standard Deviation of Net Scores Versus Handicap Index

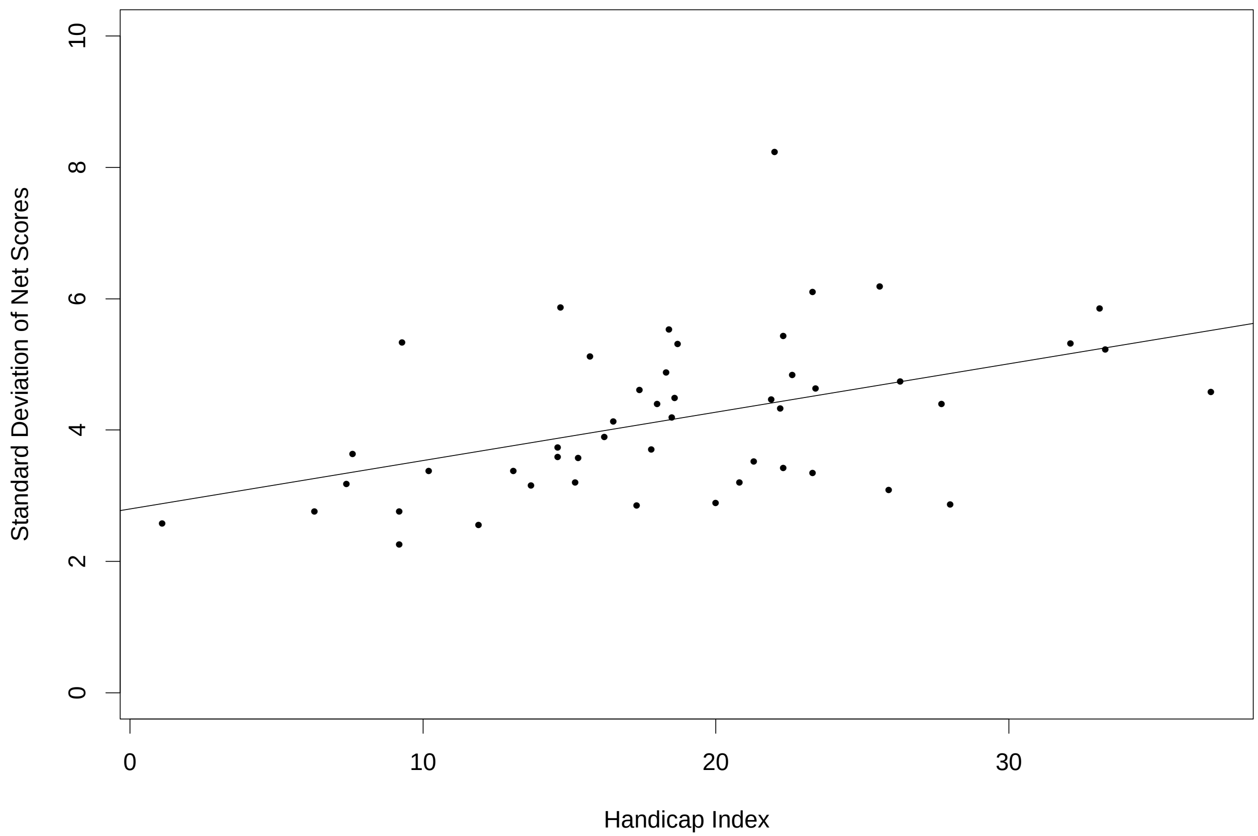

Figure 3: Standard Deviation of Differentials Versus Handicap Index

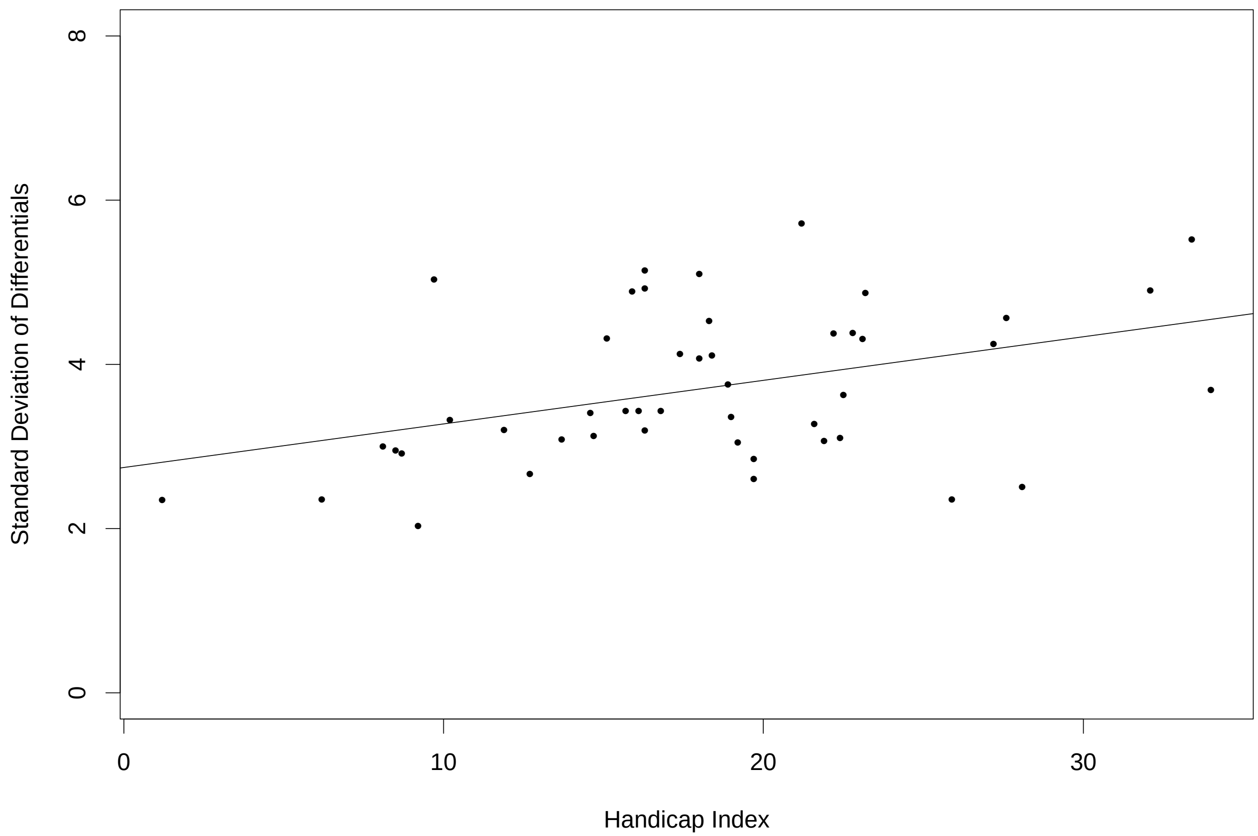

Figure 4: Minimum T* Versus Handicap Index