Acceleration and Age in Soccer

Tianyu Guan and Tim B. Swartz *

Abstract

This paper considers how player acceleration changes in soccer relative to age. A plot of average maximum acceleration versus age is produced. The construction of the plot is based on methods from functional data analysis and the availability of tracking data from the 2019 season of the Chinese Super League. For an individual player, we calculate his maximum acceleration for each single match of the 2019 season. Since the players' maximum accelerations are observed only on a single season instead of their entire careers, we treat them as incomplete functional data, called functional snippets. The average maximum acceleration, i.e., the mean function of the functional snippets rather than full curves is estimated by a local linear smoothing method. The most important observation is that the shape of the acceleration curve closely resembles curves of soccer performance versus age. This observation has implications for predicting future performance since acceleration is more easily and more accurately measured than performance.

Keywords: acceleration, functional data analysis, performance analysis, soccer analytics, tracking data.

*T. Guan is Assistant Professor, Department of Mathematics and Statistics MCJ415, Brock University, 1812 Sir Isaac Brock Way, St. Catharines, Ontario L2S3A1, and T. Swartz is Professor, Department of Statistics and Actuarial Science, Simon Fraser University, 8888 University Drive, Burnaby BC, Canada V5A1S6 (Email: tim@stat.sfu.ca). Guan and Swartz have been partially supported by the Natural Sciences and Engineering Research Council of Canada. The work has been carried out with support from the CANSSI (Canadian Statistical Sciences Institute) Collaborative Research Team (CRT) in Sports Analytics. The authors thank Daniel Stenz, former Technical Director of Shandong Luneng Taishan FC who provided the data used in this paper. The authors also thank the Reviewer László Csató whose helpful comments improved the paper.

1 INTRODUCTION

In sport, the term "quick" is frequently used to describe athletes. If an athlete is deemed quick in team sports, it is generally regarded as an advantage. By definition, quickness is a measure of speed. However, we believe that many sports insiders regard quickness more as a measure of acceleration. For example, in many team sports, changing direction is important, and this is a trait that is measured via acceleration.

The investigation considered in this paper was motivated by a hunch: namely, that maximum player acceleration in soccer is closely related to player performance. This hypothesis is potentially of great interest since the prediction of future player performance in soccer is a fundamental problem facing teams. Naturally, teams wish to build successful rosters, and this is addressed via player retention and acquisition. One aspect in the prediction of future player performance that is well-known is the impact of aging; young players improve, reach a plateau and then decline in performance. However, although the average shape of the performance/aging curve is concave, there is variation in player-specific performance/aging curves. For example, some players reach their peak quickly/slowly, and other players have rapid/gradual declines beyond their peak. Another challenging aspect involving player-specific performance/aging curves is that performance is difficult to measure. A player's performance can be impacted by various factors including luck and teammates. If it is the case that maximum acceleration/aging curves closely resemble performance/aging curves, then maximum acceleration may be used as a proxy for performance. The great advantage in this case is that maximum player acceleration can be accurately and relatively simply measured. Our analysis reveals that the two curves are similar in shape, and this insight may facilitate the prediction of future player performance.

In sports analytics, there seems to be a limited literature on the relationship between maximum acceleration involving soccer specific movements and age. Lorenzo-Martinez et al. (2021) classified 420 La Liga players into four age groups, and found that older players (31-38 years of age) had fewer accelerations and decelerations per match than players 17-23 years, 24-27 years and 28-30 years. However, their findings revealed no significant age-related declines in players' maximum acceleration, and only trivial differences were observed in maximum deceleration. In sports science, there is a common research thread that investigates the relationship between acceleration and various physical measurements. See for example, Zhang et al. (2022), Loturco (2019), Yildiz et al. (2018) and Little and Williams (2005).

In this paper, we study maximum acceleration for players in soccer. During the course of a match, modern technology permits the constant measurement of a player's acceleration. However, there is often no need for the player to exhibit high-end acceleration. Therefore, we are not interested in average acceleration, but rather the upper limits that a player can accelerate, i.e., their maximum acceleration. Maximum acceleration is an intrinsic property that contributes to player ability.

To measure maximum acceleration during a match, we have access to, and we utilize player tracking data. With player tracking data, the location coordinates for every soccer player on the field are recorded frequently (e.g., 10 times per second in soccer). With such detailed data, the opportunity to explore novel questions in sport has never been greater. The massive datasets associated with player tracking also introduce data management issues and the need to develop modern data science methods beyond traditional statistical analyses. Gudmundsson and Horton (2017) provide a review of spatio-temporal analyses that have been used in invasion sports where player tracking data are available.

Ideally, we would like to observe the maximum acceleration profile for a player with respect to their age. Such data are functional data where each player's acceleration curve is unique. Techniques from functional data analysis (FDA) would then permit the construction of an average curve across all players. For a practical introduction to FDA, see Ramsay and Silverman (2005) and Ramsay, Hooker and Graves (2009).

However, our processed data for an individual soccer player takes the form of maximum acceleration observed during a match, measured across all matches during a season. Therefore, our acceleration data are truncated from a career to a single season. We treat the incomplete acceleration data as functional snippets, a type of functional data which are observed on a short segment of the entire domain. The analysis of such data requires more sophisticated methods from FDA. Specifically, we make use of a local linear smoothing method. See, for example, Lin and Wang (2022), Zhang and Wang (2016), and Yao, Müller, and Wang (2005). FDA is an important tool for sports analytics since many quantities of interest are indexed by the time of the match, by matches or by seasons. FDA has been utilized in various sports including basketball (Chen and Fan 2018) and rugby league (Guan et al. 2020). Statistical contributions to sport are highlighted by Albert et al. (2017).

In Section 2, we describe the data used in the development of the maximum acceleration curve. Section 3 is the more technical component of the paper where FDA techniques are implemented to handle the truncated acceleration data. The major result from this section is the construction of the maximum acceleration versus age curve shown in Figure 3. In Section 4, we provide an introduction to aging curves in sport, and demonstrate the remarkable similarity between the maximum acceleration curve produced in Section 3 with a soccer aging curve. The similarity has profound implications for the prediction of future performance, a problem which has been traditionally difficult to solve. In Section 5, we describe an informal procedure for assessing maximum acceleration curves. A short discussion follows in Section 6.

2 DATA

For this investigation, we have a big data problem where player tracking data are available for 237 regular season matches (three matches missing) from the 2019 season of the Chinese Super League (CSL). The schedule is balanced where each of the 16 teams plays every opponent twice, once at home and once away.

There are various ways in which tracking data can be collected. One approach involves the use of Radio-Frequency Identification (RFID) technology where each player and the ball have tags that allow for the accurate tracking of objects. In the CSL dataset, tracking data are obtained from video and the use of optical recognition software. Our tracking data consist of roughly one million rows per match where the data are recorded every 1/10th of a second. Each row corresponds to a particular player at a given instant in time.

Although the inferences gained via our analyses are specific to the CSL, the methods are applicable to any soccer league which collects tracking data. Furthermore, the CSL is the top professional league in China, and we suspect that player acceleration profiles may be similar between the CSL and other major soccer leagues. CSL tracking data have been used in various recent sport analytics investigations including Guan, Cao and Swartz (2023), Wu and Swartz (2023a), Epasinghege Dona and Swartz (2023a,b) and Wu et al. (2021).

Using the CSL tracking data, we now consider the calculation of maximum acceleration for a particular player for a single match. The following development is based on Wu and Swartz (2023b) where there is an emphasis on the accuracy of calculations involving player tracking data. If $(x(t), y(t))$ denotes the location of a player at time t, then the player's velocity at time t in the x-coordinate direction is defined by

$$v_x(t) = \lim_{\Delta \to 0} \frac{x(t+\Delta) - x(t-\Delta)}{2\Delta} \ . \tag{1}$$

Similarly, in the y-coordinate direction, the player's velocity at time t is defined by

$$v_y(t) = \lim_{\Delta \to 0} \frac{y(t+\Delta) - y(t-\Delta)}{2\Delta} \ . \tag{2}$$

Formulae (1) and (2) are mathematical expressions involving limits, and are not quantities that can be calculated from data. Instead, with tracking data, player locations are obtained at regular times. Assume the regular times are denoted by $0 = t_0 < t_1 < t_2 < \cdots < t_n$. Since the tracking data are recorded every 1/10th of a second, the unit time increment is 0.1s, i.e., $t_{j+1} - t_j = 0.1$s for $j = 0, \ldots, n-1$. Therefore, it is reasonable to approximate $v_x(t_j)$ in (1) by

$$\hat{v}_x(t_j) = \frac{x(t_j+\Delta) - x(t_j-\Delta)}{2\Delta} \tag{3}$$

and to approximate $v_y(t_j)$ in (2) by

$$\hat{v}_y(t_j) = \frac{y(t_j+\Delta) - y(t_j-\Delta)}{2\Delta} \tag{4}$$

where $\Delta = 0.4$ seconds is recommended for accuracy by Wu and Swartz (2023b).

We measure acceleration as a scalar quantity, without a directional component. Therefore, following the above development, and using (3) and (4), we approximate $a(t_j)$, the acceleration at time t_j by

$$\hat{a}(t_j) = \frac{\sqrt{(\hat{v}_x(t_j + \Delta) - \hat{v}_x(t_j - \Delta))^2 + (\hat{v}_y(t_j + \Delta) - \hat{v}_y(t_j - \Delta))^2}}{2\Delta} \ . \tag{5}$$

Finally, we scan all of the player's acceleration calculations (5) in a match, and obtain the player's maximum match acceleration

$$a_{\max} = \max_j \hat{a}(t_j) \ . \tag{6}$$

As an alternative estimate to $a_{\max}$ in (6), one may consider selecting a player's highest recorded accelerations, and averaging these values.

Ideally, we would like to record $a_{\max}$ for every player in the CSL. However, maximum acceleration is not exhibited by every player, especially if they played abbreviated minutes. We therefore restricted data collection to players who played sufficiently often. For our analysis, we calculated maximum acceleration $a_{\max}$ for a player in a match only if he played at least 30 minutes. We then restricted the dataset to those players who had at least 20 observations (i.e., at least 20 matches where they had played at least 30 minutes). There were 109 such players in our dataset which excludes goalkeepers. The 109 players consist of 39 defenders, 46 midfielders, and 24 forwards. In Figure 1, we provide a barplot that illustrates the ages of the 109 players based on their age in their first match. There is only one player who is under the age of 20, and there are 39 players beyond the age of 30. Later in Section 4, we present the maximum acceleration versus age curve in Figure 3. We note that the shape of the curve is not sensitive to minor deviations in the data restriction parameters (i.e. 30 minutes and 20 matches) described above.

We now illustrate the data collected for a particular player. Consider Paulinho from the Guangzhou Evergrande Taobao Football Club. Paulinho has enjoyed a distinguished

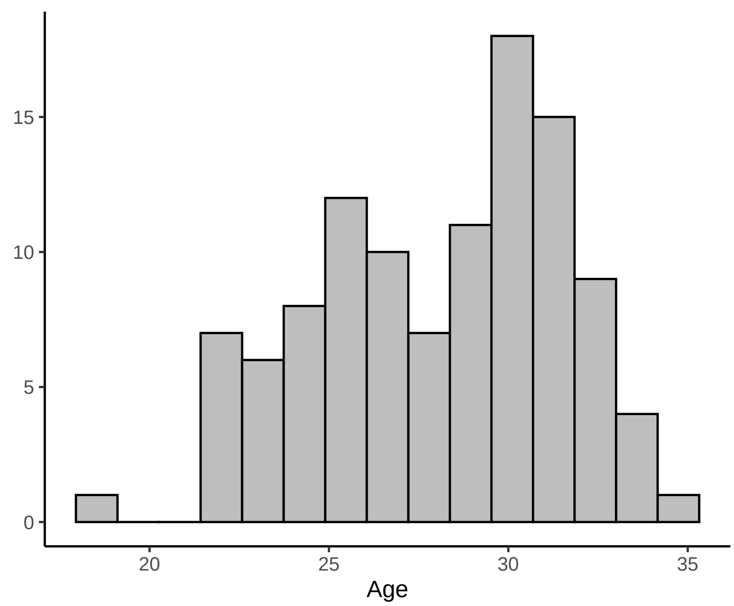

Figure 1: Barplot of the ages of the 109 players in the dataset.

football career having played internationally for Brazil (2011-2018) and for Tottenham Hotspur in the English Premier League (2013-2015). In Figure 2, we plot Paulinho's maximum acceleration over his 28 matches. With age plotted on the horizontal axis, we note that Paulinho began the season at 30.60 years of age, and finished the season at 31.35 years of age. Paulinho had an average $a_{\max}$ score of 8.83 metres/sec^2 over the season.

3 CONSTRUCTION OF ACCELERATION CURVES

Let $A(x)$ denote a smooth random maximum acceleration function defined on an age interval $\mathcal{I}$ with unknown mean function $\mu_{a_{\max}}(x) = E(A(x))$. Assume we have N players and let $A_1(x), \ldots, A_N(x)$ be their maximum acceleration functions, which are N independent samples of $A(x)$. For the ith player, instead of observing their entire career's maximum accelerations, we observe $A_i(t)$ only on a short subinterval of $\mathcal{I}$. Specifically, we assume that $A_i(x)$ is observed at m_i age levels, $x_{i1} < x_{i2} < \cdots < x_{im_i}$. Let $a_{\max ij}$ be the observed

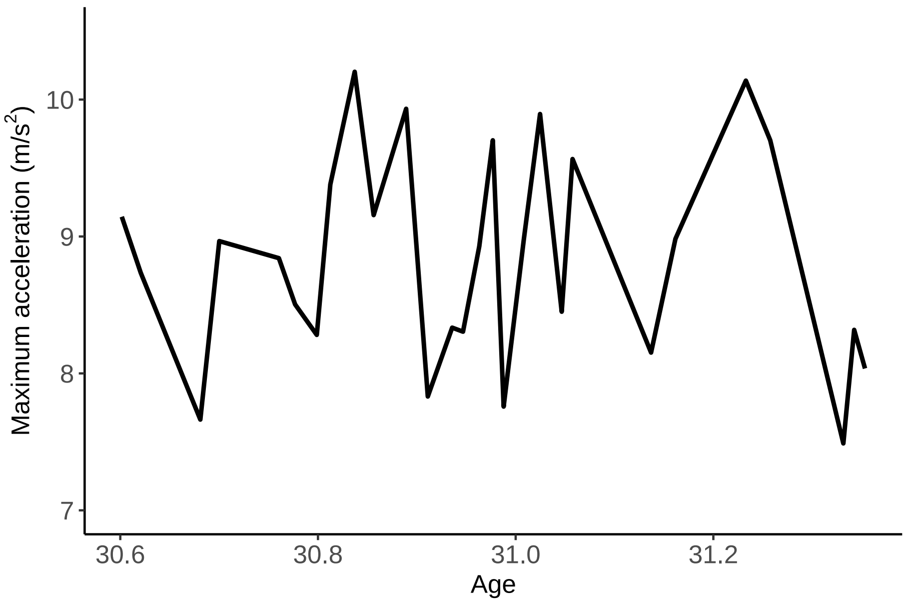

Figure 2: Plot of $a_{\max}$ for Paulinho during his 28 matches of the 2019 CSL season.

maximum acceleration of player i at age x_{ij}, $i = 1, \ldots, N$ and $j = 1, \ldots, m_i$. Then the observed data $a_{\max ij}$ is modeled as

$$a_{\max ij} = A_i(x_{ij}) + \epsilon_{ij}, \quad j = 1, \ldots, m_i,$$

where ϵ_{ij} are iid random errors with zero mean and constant variance.

We use a ridged version of the local linear smoother proposed by Lin and Wang (2022) to estimate the mean function $\mu_{a_{\max}}(x)$ of the maximum acceleration functional snippets. We first introduce the nonridged local linear smoothing estimator $\tilde{\mu}_{a_{\max}}(x)$. Let K be a one-dimensional kernel function and h be the associated bandwidth. Denote $K_h(u) = K(u/h)/h$. Then the nonridged local linear smoothing estimator $\tilde{\mu}_{a_{\max}}(x) = \hat{\beta}_0$ where $\hat{\beta}_0$ is obtained by minimizing

$$\sum_{i=1}^{N} w_i \sum_{j=1}^{m_i} \left[a_{\max ij} - \beta_0 - \beta_1(x_{ij} - x)\right]^2 K_h(x_{ij} - x) \tag{7}$$

with respect to β_0 and β_1 for each x. In (7), the w_i are nonnegative weights that satisfy $\sum_{i=1}^{N} m_i w_i = 1$. We can either assign the same weight to each maximum acceleration observation using $w_i = 1/\sum_{i=1}^{N} m_i$ or assign the same weight to each player using $w_i = 1/(Nm_i)$. In this paper, we choose the first scheme as it assigns the same importance to each game which is consistent across players. For more details regarding the two weighting schemes and optimal choice of w_i, readers are referred to Zhang and Wang (2016).

Minimizing (7) yields the nonridged estimator

$$\tilde{\mu}_{\max}(x) = \hat{\beta}_0 = \frac{p_2(x)q_0(x) - p_1(x)q_1(x)}{p_0(x)p_2(x) - [p_1(x)]^2} \tag{8}$$

where

$$\begin{aligned} p_k(x) &= \sum_{i=1}^{N} w_i \sum_{j=1}^{m_i} K_h(x_{ij} - x)\,(x_{ij} - x)^k\,, \quad k = 0, 1, 2 \\ q_k(x) &= \sum_{i=1}^{N} w_i \sum_{j=1}^{m_i} K_h(x_{ij} - x)\,(x_{ij} - x)^k\, a_{\max ij}, \quad k = 0, 1\ . \end{aligned}$$

To avoid a zero denominator in (8), a small positive constant ε is introduced which leads to the ridged local linear smoothing estimator

$$\hat{\mu}_{a_{\max}}(x) = \frac{p_2(x)q_0(x) - p_1(x)q_1(x)}{p_0(x)p_2(x) - [p_1(x)]^2 + \varepsilon 1_{\{|p_0(x)p_2(x) - [p_1(x)]^2| < \varepsilon\}}}\ .$$

A simple choice for ε is $\varepsilon = (N\overline{m})^{-2}$ with $\overline{m} = N^{-1}\sum_{i=1}^{N} m_i$.

In the estimating procedure, we use the Gaussian kernel function K and select the bandwidth h by a fivefold cross-validation. In Figure 3, we provide the estimated mean maximum acceleration curve $\hat{\mu}_{a_{\max}}(x)$ based on the ridged local linear smoothing method along with the 75% point-wise bootstrap confidence interval (CI). We observe that average $a_{\max}$ increases until about 24 years of age. This is intuitive as young players train and become stronger. Average maximum acceleration then appears to plateau until approxi-

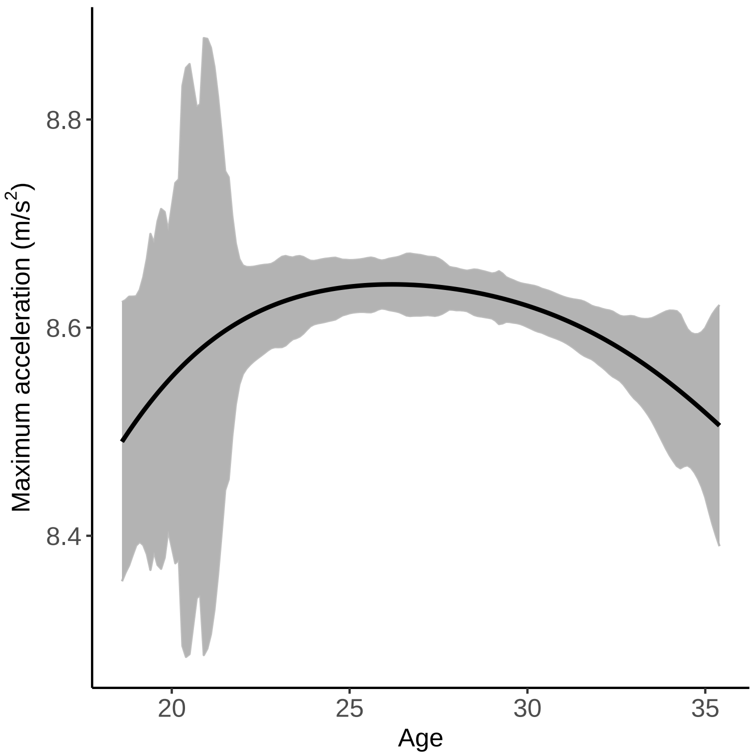

Figure 3: Curve and 75% CI of maximum acceleration versus age obtained from the FDA methods of Section 3.

mately 28 years of age. After 28 years of age, average maximum acceleration declines, and diminishes faster in subsequent years. We also note that estimation is more variable for young players less than 22 years of age. This is in keeping with Figure 1 where we observe few young players in the dataset.

We remind the reader that the mean maximum acceleration curve in Figure 3 is a spline function based on FDA methods. By eye, the curve does seem to have a nearly quadratic shape and this simple approximation may be useful to practitioners. Using the points from Figure 3 at half year increments, we fit the model $\hat{\mu}_{a_{\max}}(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \epsilon$ and obtain estimates $\hat{\beta}_0 = 7.236972$, $\hat{\beta}_1 = 0.104893$, $\hat{\beta}_2 = -0.001954$ and the fit diagnostic $R^2 = 0.982$.

4 RELATIONSHIP TO AGING CURVES

Important decisions made by the front office staff of major sports clubs involve predicting the future performance of players. These decisions typically arise in the context of drafting, player retention and player acquisition. Good decisions naturally rely on accurate prediction models that relate performance to age. The development of such models in team sports is challenging due to the fact that player performance is highly dependent on teammates and the number of minutes played. This area of research is further complicated by the fact that individual athletes age differently. The resultant curves that relate performance to age are known as aging curves. On average, players tend to improve from their earliest seasons, plateau, and then decline in performance until they retire from their sport.

There has been considerable work done on aging curves in sport. In soccer alone, aging curves have been developed by Swartz, Arce and Parameswaran (2013), Dendir, S. (2016) and Kalén et al. (2019). Other contributions that address aging in soccer include Sal de Rellán-Guerra et al. (2019), Jamil and Kerruis (2020), Rey et al. (2022, 2023) and García-Calvo et al. (2023). A review of aging curves and a new approach for their construction is given by Cavan, Cao and Swartz (2023).

Swartz, Arce and Parameswaran (2013) produce an aging curve for soccer that generally resembles alternative constructions of aging curves. An interesting aspect of Swartz, Arce and Parameswaran (2013) is that they utilize salary data to define the required player performance measure. They suggest that team personnel (who determine salaries) have an intimate knowledge of player performance that is perhaps superior to contextual statistics that may also be used to gauge player performance. With access to the aging curve data developed by Swartz, Arce and Parameswaran (2013), we denote their performance measure $p(x)$ which is a function of player age x.

To compare the acceleration curve of Section 3 with the aging curve of Swartz, Arce and Parameswaran (2013), we estimate parameters from the simple linear regression model for performance

$$p(x) = b_0 + b_1 \mu_{a_{\max}}(x) + \epsilon(x) \tag{9}$$

where $\epsilon(x)$ is the random process. The linear regression scales the maximum acceleration curve from Figure 3 so that it can be easily compared to the aging curve.

In (9), we obtain least square estimates $\hat{b}_0 = -2076.87$ and $\hat{b}_1 = 249.22$. However, the most stunning revelation from the linear regression output is that the sample correlation coefficient between $p(x)$ and $\mu_{a_{\max}}(x)$ is 0.991.

Dendir (2016) presents various analyses that address peak age of soccer players. In particular, Dendir (2016) considers separate analyses for defenders, midfielders and forwards. We calculated the sample correlation between our maximum acceleration curve (Figure 3) and the fixed effect quadratic regressions of Dendir (2016). Again, we obtained strong correlations of 0.822, 0.908, and 0.802 in the case of defenders, midfielders and forwards.

In Figure 4, we superimpose the maximum acceleration curve of Figure 3 with the aging curve of Swartz, Arce and Parameswaran (2013). The agreement between the curves is very strong, and this leads to an implication of potentially great importance. Namely, since performance is difficult to assess and maximum acceleration is easy to measure, teams may consider using maximum acceleration as a proxy for performance. When a player's maximum acceleration begins to decline, this may be a signal of anticipated reduced performance.

5 PERFORMANCE PREDICTION IN PRACTICE

In practice, suppose that we have a player for whom 5-10 years of tracking data are available. Although formal methods for assessing future performance may be developed, we describe a simple informal procedure that may be sufficient for front office staff.

For each match in the 5-10 year window, we calculate the player's maximum acceleration $a_{\max}$ given by (6). For simplicity, we may then average the $a_{\max}$ values over a season and denote $a_{\max_a}$ as the player's average maximum acceleration at age a where a is the player's age at the midpoint of the season. We then plot $a_{\max_a}$ versus a, and compare these points to the maximum acceleration curve given in Figure 3. It may not be the case that the

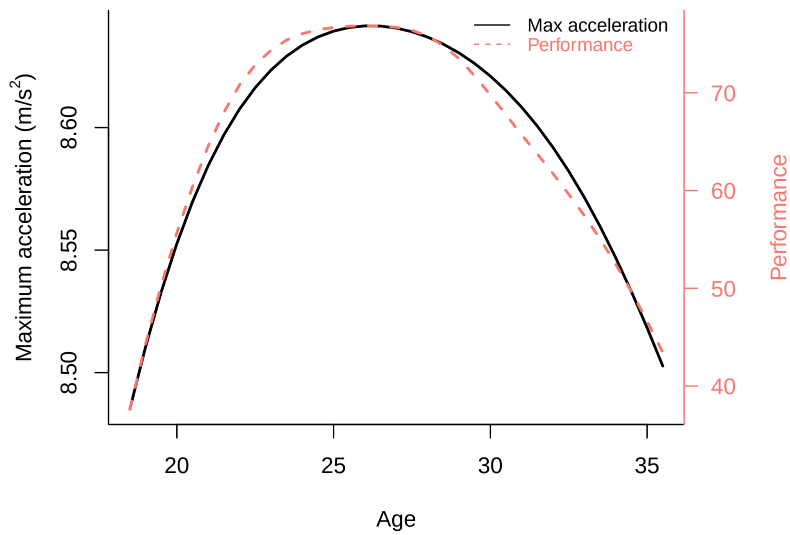

Figure 4: Plot of maximum acceleration curve of Figure 3 superimposed with the scaled soccer aging curve from Swartz, Arce and Parameswaran (2013).

points align perfectly with the smooth curve; for example, the player may have been an early/late bloomer. What is important to observe is whether a decline has begun in the points and whether the decline is gradual/steep. A steep decline indicates that the player may not have much left in their career.

6 DISCUSSION

This paper contructs a plot of maximum acceleration versus age in soccer. The approach is based on methods from functional data analysis and utilizes tracking data from the 2019 season of the Chinese Super League. We have argued that maximum acceleration is an intrinsic characteristic of players. Therefore, the variability of maximum acceleration for a player during the course of a season should perhaps not be as great as shown (see for example, Figure 2). The variability which we observe may be due to the fact that during particular games, players do not need to accelerate to their maximum ability. Going forward, we might reduce the variation in a player's plot by calculating maximum acceleration from a group of consecutive games rather than a single game. Alternatively, teams could measure maximum acceleration from training sessions where specific movements are encouraged to obtain more precise measurements of maximum acceleration.

Although the technical development of the acceleration plot and the resultant curve are interesting in their own right, the similarity between the acceleration plot and aging curves in soccer has potentially great impact for decision making in sports analytics. We have argued that the assessment of performance relative to age is a tremendously important problem for professional sports teams. An issue with performance is that performance is difficult to assess; did the player not score because of bad luck or is the player really experiencing a decline in performance due to age? On the other hand, maximum acceleration can be measured easily, and with minimal error. We also note that match performance metrics typically depend on a player's team and teammates; this is not the case with the maximum acceleration metric. Acceleration can also be measured by teams over seasons. Therefore, changes in maximum acceleration with respect to age can be detected. If, as it seems, maximum acceleration is highly correlated with performance, then maximum acceleration may be used as a proxy by teams to predict future performance.

As pointed out by the Reviewer, in the future, it is conceivable that teams may consider maximum acceleration as a proxy for the prediction of performance, which creates incentives for players to improve maximum acceleration. This insight suggests various questions: Is it possible to markedly improve maximum acceleration via training? If so, would the observed strong correlation between maximum acceleration and performance continue to exist?

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