

An Introduction to Control Charts

February, 2009

Control Charts

The basics

- Control charts are used to monitor and/or improve a process.
- We will develop methods to monitor the mean and variability of an iid, normally distributed characteristic, X .
- Use \bar{x} chart to monitor the mean
- Use R chart to monitor the variability

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The basics

Control charts generally consist of

- Data points corresponding to the measurement of a characteristic of interest over time
- Centre line - a line that represents the average value of the characteristic (when the process is in control)
- One or more upper and lower control limits - horizontal lines that help gauge whether or not the process is in control

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Defining process control

- A process is said to be in statistical control if it operates with only chance causes of variation.
- Chance causes of variation determine the inherent variability that exists in a system, no matter how well designed.
- Other sources of variation are called assignable causes, and the effects of these may be reduced or eliminated. E.g.,
 - ▶ improperly adjusted machines
 - ▶ operator errors
 - ▶ defective raw material
- A process is said to be out of control when it operates in the presence of assignable causes.

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The basics

- Suppose $X_i \sim N(\mu, \sigma_x^2)$ and X_1, \dots, X_m are independent
- Letting $\sigma = \frac{\sigma_x}{\sqrt{m}}$, $\bar{X} \sim N(\mu, \sigma^2)$ and

$$P[\bar{X} \in (\mu - z_{\alpha/2}\sigma, \mu + z_{\alpha/2}\sigma)] = 1 - \alpha.$$

- Even if X is not normally distributed, by the CLT, \bar{X} may still be approximately normally distributed. (Use QQ-plots to check.)

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Construction of control chart for the mean

- Compute μ and σ based on historical data collected when the process was operating under only chance causes
- The centre line is given by μ
- Construct UCL and LCL based on distribution of \bar{X} : prob. that $|\bar{X}|$ exceeds $\mu \pm 3\sigma$ is small if the distribution of \bar{X} remains constant
- Points outside the CLs suggest that an assignable cause is operating
- Can also construct 2σ warning limits

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There's more than one way to lose control

There are two ways a process may be deemed out of control:

- Data points plotted outside the control limits
- A non-random pattern of data points, e.g.
 - ▶ a run of monotonic data points
 - ▶ cyclic behaviour
 - ▶ step changes
 - ▶ autocorrelation
 - ▶ too many data points on one side of the centre line
 - ▶ too many data points outside the warning limits

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Decision rules for control

Montgomery (2001) suggests the following list of decision rules for assessing the state of a process:

- 1 or more points outside the control limits
- 2 or 3 consecutive points outside the 2σ warning limits but still inside the control limits
- 4 or 5 points on one side of the centre line and beyond the 1σ limits
- a run of 8 consecutive points on one side of the centre line
- 6 points in a row steadily increasing or decreasing

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Characterizing patterns

- 15 points in a row that stay within the 1σ limits
- 14 points in a row alternating up and down
- 8 points in a row on both sides of the center line with none within the σ limits
- an unusual or non-random pattern in the data
- one or more points near a warning or control limit

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Type I and type II errors

- Type I error occurs when we conclude the process is out of control when it is not (a false alarm)
- Type II error occurs when we conclude the process is in control when it is not (a lost opportunity)

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Using too many decision rules

Using too many simultaneous decision rules requires care and could result in dramatic increase in our overall Type I error.

- Suppose we use k decision rules and that criterion i has Type I error probability α_i .
- Assume the decision rules are independent (usually not true in practice!)
- Then the overall Type I error (false alarm probability) for the decision based on the k rules is $\alpha = 1 - \prod_{i=1}^k (1 - \alpha_i)$.

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Type I and type II errors

- Tight control limits (i.e. a small value of $\sigma = \frac{\sigma_x}{\sqrt{m}}$) and frequent samples improve the chances of detecting a shift in μ
- ⇒ Ideally, take large samples frequently
- But, we must consider how to allocate limited resources
 - One way is to specify our desired probability of Type I error
 - This is called setting α -probability limits.

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Using multiple control limits

- Some manufacturers use multiple control limits to increase sensitivity (1σ and 2σ warning limits, most commonly)
- Can make the process more dynamic by taking more frequent samples when a point falls in a warning zone