

NAME:  
Student Number:

## STAT 450 Midterm Examination

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25 October 2017

**Instructions:** This is an open book exam. You may use notes, books, a calculator and your computer to read my notes, the textbook or other books or notes on line. You may not use sites like Wolfram alpha or make contact with other people. The exam is out of 30. You should have 8 pages: a cover page, 5 pages of questions, a page of extra space and a grade sheet. The algebra in question 1d is probably too challenging for this exam; finish all the other parts of the exam before you try to do this part. You can get most of the marks in 1d without finishing the calculus and algebra – but you need to set everything up properly and do as much as you can. **DON'T PANIC.**

1. Suppose

$$f(x, y) = \left(kx + \frac{y}{2}\right) 1(0 < x < 1)1(0 < y < 1)$$

is the density of some pair of random variables  $(X, Y)$ .

(a) Find  $k$ .

[4 marks]

(b) Let  $F$  denote the cdf of  $X + Y$ . Find  $F(u)$  for  $0 < u < 1$ .

[2 marks]

(c) For  $1 < u < 2$  write  $1 - F(u)$  as a double integral with correct limits of integration.  
[2 marks]

(d) Find the density of  $X + Y$ . WARNING: do not spend a long time on the algebra;  
it is worth no more than 0.5 marks out of 30. [2 marks]

2. Suppose you observe  $n$  independent random variables  $X_1, X_2, \dots, X_n$ . Suppose that  $X_i$  has density

$$f_i(x_i) = \theta_i e^{-\theta_i x_i} 1(X_i > 0).$$

Finally suppose that  $\log(\theta_i) = \beta t_i$  for known constants  $t_1, \dots, t_n$  and an unknown parameter  $\beta$ .

- (a) What is the likelihood function? [3 marks]

- (b) What is the log-likelihood function? [3 marks]

- (c) Find the score function. [3 marks]

(d) Find the Fisher information.

[3 marks]

(e) What equation would you need to solve to find the mle of  $\beta$ ?

[3 marks]

- (f) Prove that for any data set  $(t_1, X_1), \dots, (t_n, X_n)$  such that every  $t_i > 0$  and there is a unique MLE  $\hat{\beta}$ . [NOTE: you don't need to assume every  $t_i > 0$  but the problem might be a bit easier if you do.] [4 marks]

Extra space

Grade Sheet Stat 450 Midterm Fall 2017

Name:

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1a		4	1b		2
1c		2	1d		2
2a		3	2b		3
2c		3	2d		3
2e		3	2f		4
Bonus	1	1			

Total		30
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