

Name: _____

Student Number: _____

STAT 380: Spring 2018

Final Examination

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15 April 2018

Instructions: This is a closed book exam. You are permitted to use 6 sheets of notes, machine-written or hand-written. You may use both sides of the sheets and I place no limits on font size. Calculators are not permitted nor are any other electronic aids. You should simplify answers as much as possible but things which would need a calculator to do can be left in the simplest form you can manage. The exam is out of **45**. You should have a total of **12** pages; this cover page has the grade sheet and the last two pages are extra space. I will be marking for clarity of explanation as well as correctness. Without a clear explanation you should not expect to get more than half marks. You have 3 hours.

1a		3	1b		3	1c		3
2a		3	2b		3	2c		3
2d		3	2e		3	2f		3
2g		3	2h		3	3a		3
3b		3	3c		3	3d		3

Total		45
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1. Particles pass through a detector at the times of a Poisson Process with a rate of 10 particles per hour. The detector does not detect all the particles; each time a particle goes through there is a $1/5$ chance that the particle is detected.

(a) What is the chance that 3 particles will be detected in the first half hour? [3 marks]

(b) Given that 2 particles are detected in the first 30 minutes what is the chance that a total of 6 particles passed through the detector? [3 marks]

- (c) Given that 1 particle is detected in the first 30 minutes what is the probability that a total of 2 particles passed through the detector in the first hour? [3 marks]

2. As in the previous problem particles pass through a detector at the times of a Poisson Process with a rate of 10 particles per hour. This time the detector has three states. When you turn it on it is in the low sensitivity state. In this state each particle is detected with probability $1/5$. When the detector is in this low sensitivity state and it detects a particle it changes immediately to a medium sensitivity state where it detects particles with chance $3/5$. In that state it waits till it first detects a particle then switches to a high sensitivity state where no particles go undetected. When a particle is detected in the high sensitivity state it goes immediately back to the low sensitivity state, and so on.

(a) Define a continuous time Markov Chain for the state of the detector. [3 marks]

(b) For the process you have just described specify the elements of the infinitesimal generator \mathbf{R} , and the transition matrix of the skeleton chain. [3 marks]

(c) In the long run what fraction of the time does the chain spend in each state? [3 marks]

(d) In the long run how many particles are detected per hour? [3 marks]

(e) In the long run what fraction of the particles are detected?

[3 marks]

(f) Does the skeleton chain have a stationary distribution? If so what is it? [3 marks]

(g) Is the skeleton chain aperiodic? If it has a period more than 1, what is it? [3 marks]

- (h) After lunch I come back to discover that the detector is in the medium sensitivity state. How long should I expect to wait until it is next in the low sensitivity state? [3 marks]

3. In a certain casino there are 3 games, A, B, and C. I start out playing A and win with probability $1/5$ every time I play. I bet a dollar each time and if I win I get the dollar back and 4 more. Every time I win I toss a fair coin. If it comes up heads I move to game B; otherwise I continue to play game A. Game B has a $1/10$ chance of winning. I bet a dollar each time and if I win I get the dollar back and 8 more. Again, when I win I toss a fair coin. If it comes up heads I move to game C; otherwise I continue to play game B. Game C has a $1/20$ chance of winning. I bet a dollar each time and if I win I get the dollar back and 8 more. Again, when I win I toss a fair coin and go back to playing A if I win.

(a) Define a suitable discrete time Markov Chain for this gambling scheme. [3 marks]

(b) What is the transition matrix for the chain? [3 marks]

(c) In the long run, on what fraction of the bets I make am I playing game C? [3 marks]

(d) In the long run, how much money do I lose per dollar bet? [3 marks]

Extra Space

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