STAT 380

Assignment 7

1. Page 399, number 5. There is a population of N individuals; some are infected with disease which spreads as follows. Contacts between any two individuals occur at times of a Poisson Process with rate λ. If the population size is n then a contact is equally likely to be any of the n choose 2 possible pairs. If one is infected, and the other is not, then the uninfected person is infected with probability p. Infected people stay that way. If $X(t)$ is the number infected at time t is it a Markov Chain? What type? What is the expected time till everyone is infected starting from 1 individual?

2. Page 399, number 2. You have two machines with exponentially distributed lifetimes. Machine i has rate μ_i of breaking down. There is a single repair person. Repair times are exponential with rate μ. Can we use a birth and death process to model this? If so, what are the parameters? If not, how can we analyze the system?

3. Page 399, number 8. You have two machines with exponentially distributed lifetimes, each with rate λ. There is one repair person; services machines are exponentially distributed with rate μ. Identify the natural continuous time Markov chain. Set up the Kolmogorov backward equations. Do not solve them.

4. **This problem is cancelled.** Page 400, number 10. You have two machines with exponentially distributed lifetimes. Machine i has rate μ_i of breaking down. Machine i has an exponentially distributed repair time with rates λ_i; two machines can be repaired simultaneously. Describe a suitable 4 state Markov Chain. Compute transition probabilities for this chain using independence of the two machines. Check the solutions solve the Kolmogorov equations.

5. Page 400, number 12. In a population each *individual* has a birth rate λ and death rate μ. Immigration occurs at rate θ but is not allowed if the population size is N or larger. Describe a suitable birth and death model and find the proportion of time that immigration is restricted if $N = 3$, $\lambda = \theta = 1$ and $\mu = 2$.

6. Page 401, number 18. A machine functions (after repair) for an exponential time with rate λ. When it breaks it goes through a k step repair process with step i taking exponential time with rate μ_i; all these times are independent. What proportion of the time is the machine undergoing step i repair and what proportion of time is it working?