

STAT 380

Assignment 5

1. Imagine that while a spacecraft is in orbit it is hit by micrometeorites at the times of a Poisson process with a rate of λ per second. If the spaceship is in orbit for T seconds what is the chance that it is not hit by any micrometeorites.
2. You and a friend arrive at a customer service counter to find there are two lines; each line has 1 person in it. Line 1 is being served by someone who takes an exponentially distributed amount of time to serve a customer with rate λ_1 per unit time. For Line 2 the service times have an exponential distribution with rate λ_2 . All the various service times are independent. If you join Line 1 and your friend joins Line 2 what is the chance that you will start being served before your friend starts being served and what is the chance that you will finish being served before your friend?
3. With the two lines functioning as in the previous question imagine that you arrive (alone this time) to find line one empty and 1 person in line 2. If you know the values of λ_1 and λ_2 which line should you join? The answer may depend on the relation between the two λ s.
4. Page 345, number 46. A Poisson process N is independent of a random time $T \geq 0$. Assume T has mean μ and sd σ . Part a) find $\text{Cov}(T, N(T))$. Part b) find $\text{Var}(N(T))$.
5. Suppose that for each n X_n and Y_n are independent Binomial random variables. Assume that the distribution of X_n is Binomial(n, p_n) and the distribution of Y_n is Binomial(n, q_n). Assume that $p_n = \lambda/n$ and $q_n = \theta/n$. Let $W_n = X_n + Y_n$. Compute the probability generating function of W_n , namely $\phi_n(s) = E(s^{W_n})$. Then compute

$$\lim_{n \rightarrow \infty} \phi_n(s).$$

Compare the result to the probability generating function of a Poisson random variable and interpret the result.

6. Page 346 number 49. For a rate λ Poisson process we try to stop at the last event before some fixed time T . Our strategy will be to stop

at the first event after some time s . We lose if there are no more events after s and before T or if the next event after s is not the last event before T . What is the probability of winning for s fixed? What value of s maximizes this probability? What is the maximum value?