

## STAT 380

### Assignment 4

1. Page 262 number 14. You are given 4 transition matrices and must find the classes of each chain and say which are transient and which recurrent.
2. Page 263 numbers 20 and 22. In problem 22 you assume that  $Y_n$  is the sum of  $n$  rolls of a fair die. Find

$$\lim_{n \rightarrow \infty} P(Y_n \text{ is divisible by } 13).$$

In problem 20 you must prove that if the sum of each *column* of a transition matrix is 1 then the uniform distribution on the states is a stationary initial distribution. The two problems are related and you might be wise to do problem 20 first.

3. Page 265 number 28. You have a team whose chance of winning its next game depends on outcome of the current game. If it wins the current game then the chance it wins the next is 0.8 and if it loses the current game the chance it wins the next is only 0.3. After a winning game the team goes out to dinner together 70% of the time; after a losing game only 20% of the time. What proportion of games result in the team going out to dinner?
4. Page 267 number 42. Suppose  $A$  is a set of states for some chain and  $A^c$  is the rest of the states. If  $\alpha$  is the stationary initial distribution and  $\mathbf{P}$  is the transition matrix then give interpretations of the quantities

$$\sum_{i \in A} \sum_{j \in A^c} \alpha_i P_{ij}$$

and

$$\sum_{i \in A^c} \sum_{j \in A} \alpha_i P_{ij}$$

Then explain the fact that these two are equal.

**for the next assignment.** Page 271 Number 60. Given a 4 by 4 transition matrix (states 1, 2, 3, 4)

$$\begin{bmatrix} .4 & .3 & .2 & .1 \\ .2 & .2 & .2 & .4 \\ .25 & .25 & .5 & 0 \\ .2 & .1 & .4 & .3 \end{bmatrix}$$

compute the chance that the chain enters state 3 before state 4 and the mean number of transitions until either state 3 or state 4 is entered.