“Baby” Bayes

Leanna House



Where we are going...

Bayesian Calibration

Posterior Distribution

$$\pi(\boldsymbol{\theta}, \mu, \lambda_\eta, \boldsymbol{\rho}_\eta, \lambda_\delta, \boldsymbol{\rho}_\delta | \mathcal{D})$$

$$\propto L(\mathcal{D} | \boldsymbol{\theta}, \mu, \lambda_\eta, \boldsymbol{\rho}_\eta, \lambda_\delta, \boldsymbol{\rho}_\delta, \boldsymbol{\Sigma}_y) \times \pi(\boldsymbol{\theta}) \times \pi(\mu)$$

$$\times \pi(\lambda_\eta) \times \pi(\boldsymbol{\rho}_\eta) \times \pi(\lambda_\delta) \times \pi(\boldsymbol{\rho}_\delta),$$

Likelih
or
Joint Sampling Distribution

Prior Distributions

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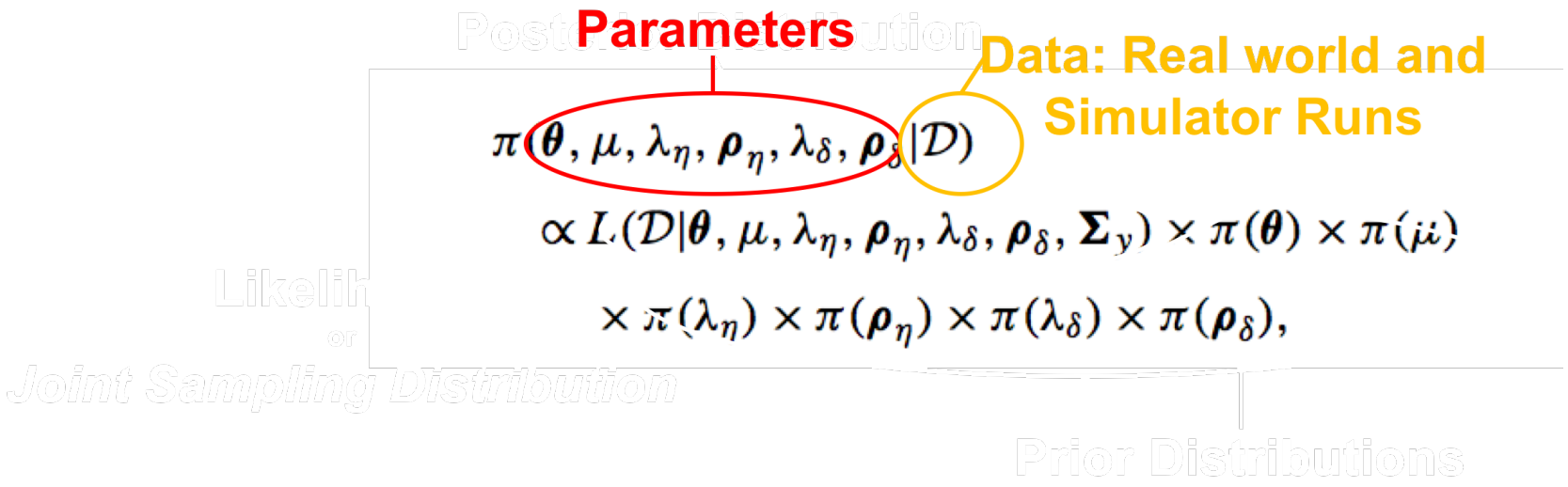
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Bayesian Calibration



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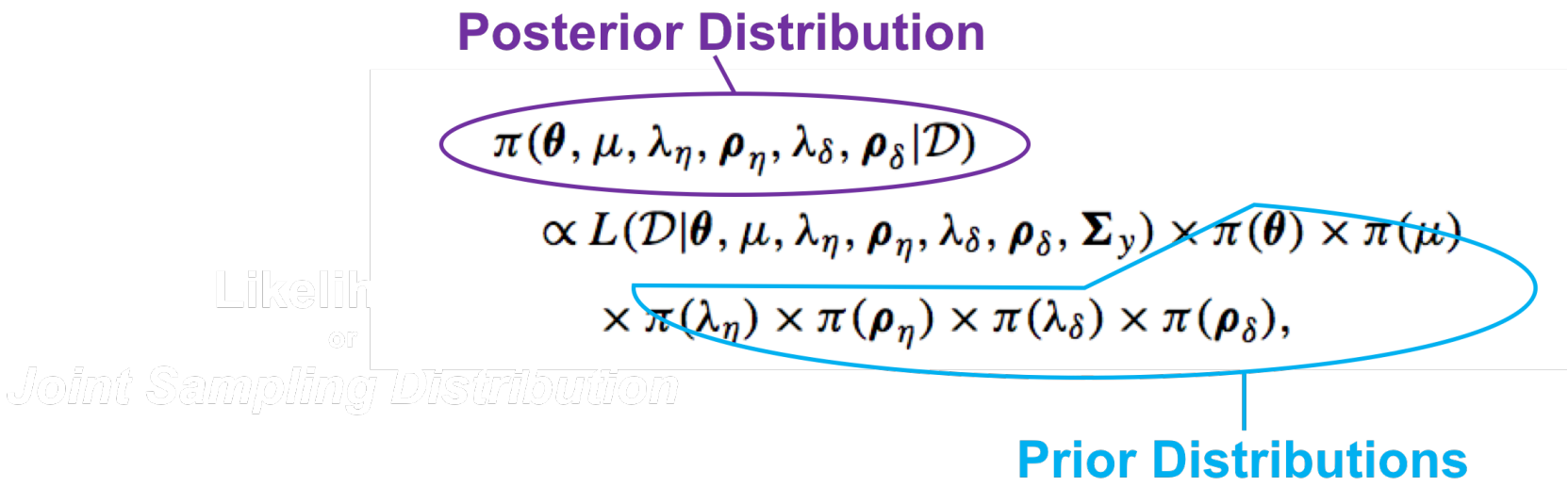
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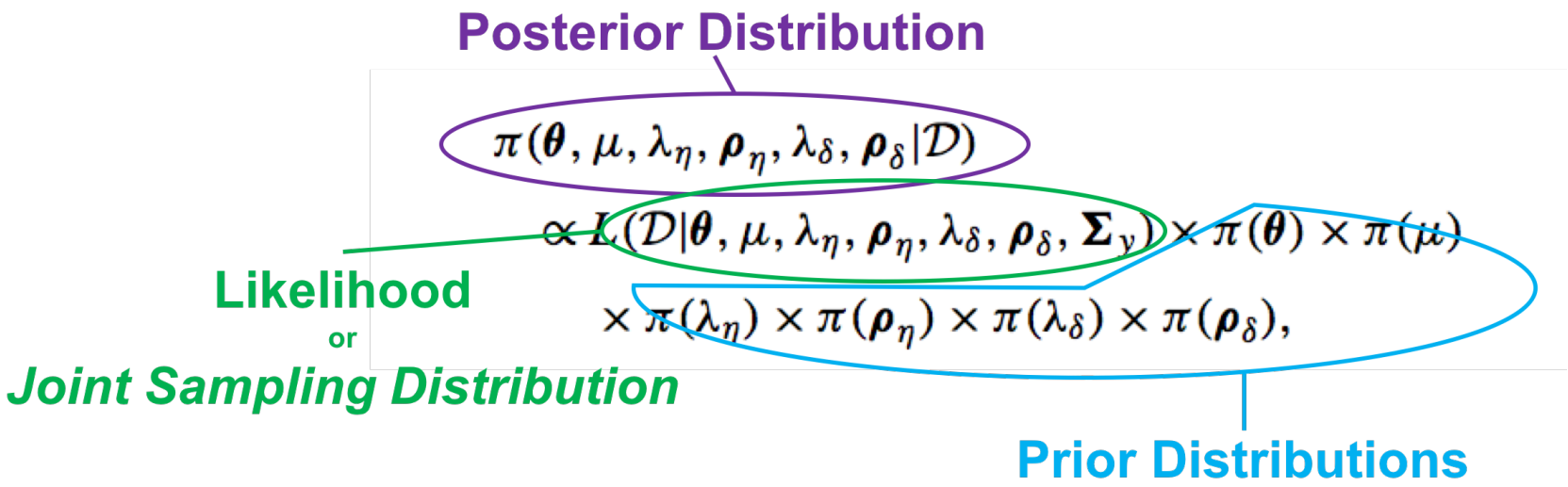
Where we are going...

Bayesian Calibration



Where we are going...

Bayesian Calibration



Outline

- What is Bayesian statistics?
- Why Bayesian statistics?
- Why *not* Bayesian statistics?
- Turn the Bayesian Crank
- Examples of Bayes
- Context in Computer Models (Simulators)

What is Bayesian statistics?

What is Bayesian statistics?

First, many other questions....

What is statistics?

What is statistics?

Methods for discovering information in data
and making inference.

What is inference?

What is inference?

Judgements about a population from which data are collected.

Judgements about what?

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Unknowns in the population: Parameters

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Unknowns in the population: Parameters

E.g., Consider the height of daisies in Blacksburg, VA...

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- Population slope between height and number of pedals, β

To make good inference,
what must we ALWAYS communicate?

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Uncertainty in our judgements

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Posterior Probability !

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$\pi(\theta|y)$ in words:

The probability distribution of parameters GIVEN data.

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Kids Understand the Probabilities of Possibilities Better Than Adults Do

By Alison Gopnik | Posted Thursday, Oct. 11, 2012, at 3:30 AM ET
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Slate.com

ENABLE SOCIAL READING Why Your 4-Year-Old Is As Smart as Nate Silver

And if kids are so smart, why are adults so stupid about statistics?



How do children learn so much? Photograph by Brand X Pictures/Thinkstock.

Everyone who spends time with children knows how incredibly much they learn. But how can babies and young children possibly learn so much so quickly? In a recent article in Science, I describe a promising new theory about how babies and young children learn and the slew of research that supports it. The idea is that kids learn by thinking like Nate Silver, the polling analyst extraordinaire at the New York Times.

I suspect that most people who, like me, obsessively click his FiveThirtyEight blog throughout the day think of Nate as a semi-divine oracle who can tell you whether your electoral prayers will be answered. But there is a very particular kind of science behind Nate's predictions. He uses what's called Bayesian modeling, named after the Rev. Thomas Bayes, an 18th-century mathematician. The latest studies show that kids, at least unconsciously, are Bayesians, too.

Why Bayesian statistics?

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- It makes sense!
  - Classical: 95% Confidence interval:

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Why *NOT* Bayesian statistics?

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Why *NOT* Bayesian statistics?

- Tradition
- Computation
- Subjective?
  - Definition of probability
  - Prior distribution influences findings from Likelihood

From this point onward....

put on your Bayesian hats.

The Bayesian Crank (Bayes Rule)

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Start: Specify a sampling distribution

What might be the distribution governing the data?

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RQ:	**Data**	**Distribution?**
Is the coin fair?	$\{H, H, T, H...H\}$	
Is the coin fair?	$\{H\}$	
Avg. height of daisies?	$\{11.1, 12.2, 3.4, ...15.1\}$	
Avg. # of cars that stop?	$\{5, 3, 15, 10, ...50\}$	

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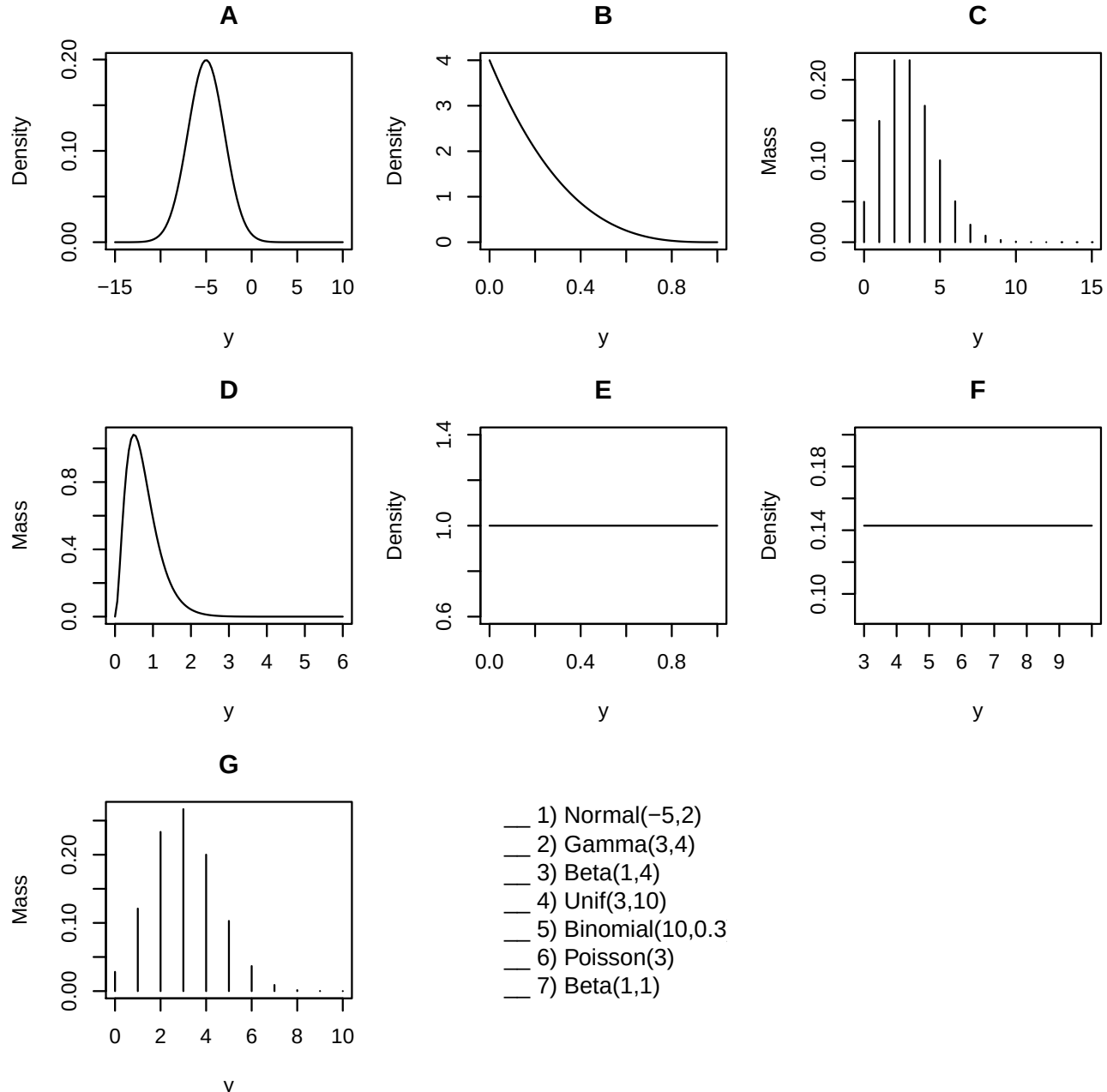
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Tangent: Common Distributions



__ 1) Normal(−5,2)
__ 2) Gamma(3,4)
__ 3) Beta(1,4)
__ 4) Unif(3,10)
__ 5) Binomial(10,0.3
__ 6) Poisson(3)
__ 7) Beta(1,1)

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Parameter:	**Data**	**Distribution?**
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Finally: Turn the Bayesian Crank

Apply Bayes Rule...

(Go to paper for examples: Bernoulli, Normal)

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Based on summaries of the posterior distribution

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- Relevant posterior probability statements: $P[\theta > c|y]$

Context: Simulators

- z: Real Data
- y: Features of a true process
- f: Simulator
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$$
\begin{aligned}
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y(t)|f,t &\sim \text{Normal}(f(t), {\phi_y}^{-1}) \\
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- θ: Ideal value for t
- ϕ_z,ϕ_y: Precisions
- $m(t)$, $p(t)$: Posterior predictive parameters for the emulator.

Thank You!