On the Misuse of Residuals in Ecology: Testing Regression Residuals vs. the Analysis of Covariance

November 29, 2011

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Introduction

- In ecology and many other disciplines, we often compare a response variable among several treatments or groups after removing the effect of a third, concomitant variable.

- Ex: Comparison of body condition among several animal population or groups.
Three Main Statistical Procedures

- To remove the size effect or concomitant variable.

1. Compute a ratio

2. The analysis of covariance (22% & 12%)

3. Residual index (8% & 2.2%)
The aim of this article

• To point out several pitfalls of using the residual index for studying condition.

• To emphasize the use of ANCOVA as the correct alternative.
What is residual index?

Definition: An analysis of variance or other linear models of the residuals of a simple linear regression is being increasingly used in ecology to compare two or more groups. Such a procedure is called “residual index”.

What is residual index?

• Regression model
• \( y_{ij} = a + b \cdot x_{ij} \)
• Residual = \( y_{ij} - a - b \cdot x_{ij} \)

• We know that \( a = \bar{y} - b \bar{x} \)
• So,
• Residual = \( y_{ij} - \bar{y} - b( x_{ij} - \bar{x}) \)
What is residual index?

- One Facotor ANCOVA

\[ y_{ij} = \mu + \gamma_i + \gamma_{within} (x_{ij} - \bar{x}) + e_{ij} \]

- \[ y_{ij} - \mu - \gamma_{within} (x_{ij} - \bar{x}) = \gamma_i + e_{ij} \]

- \[ y_{ij} - y - b(x_{ij} - \bar{x}) \]

They are similar
• Although, the residual index is similar to an analysis of covariance (ANCOVA), it is not identical and yields different statistical results for four reasons.
Reason #1

- The regression coefficient used by the residual index differs from the one used in ANCOVA.
- The “b within” in standard ANCOVA is the weighted regression coefficients for each group and least-squares estimator.
- Whereas the overall b in the linear regression is the estimator ignoring the group and should not be used if the objective is to compare the groups.
Reason #2

- In ANCOVA, error d.f are N-K-1
- In residual index, d.f are N-K
- So, compared to ANCOVA, the error degrees of freedom in the residual index is overestimated because of the estimation of regression of the coefficient.
Some numerical examples

• Sokal & Rohlf describe thoroughly an example of the standard design of ANCOVA in which the result for the factor is $F_{3,16} = 195.98$, $P<0.0005$. An ANOVA of residual index for the same data set would result in $F_{3,17} = 39.6$, $P<0.0005$.

• An artificial data set for which the two procedures give opposite conclusions.
  - $(X,Y)$ (1,2)(3,3)(5,4)(7,4)
  - $(X,Y)$ (11,12)(13,13)(15,14)(17,14)
  - ANCOVA....................... $P=0.33$
  - Residual index.............. $P<0.0005$
Reason #3

• Another advantage of ANCOVA is its flexibility in implementing any aspect of the experiment design. For example, a special design with the factor * covariate interaction allows us to test the assumption of homogeneity of regression coefficients (parallelism assumption), which is essential to the standard ANCOVA. The residual index also assumes the homogeneity of slopes, without realizing it.

• A hypothetical example will show that the residual index can lead to incorrect biological conclusions if this assumption is not satisfied.
An Example

Fig. 1. Hypothetical example of a study of condition with heterogeneity of slopes. Top: weight–length data. The length values are 2, 3, 4, and 5 and the weight values are 2, 3, 4, and 5 for the control and 1, 3, 5 and 8 for the treatment group. The variables are supposed to have been log-transformed to fit the conventional weight–length relationship ($Y = aX^b$). Simple linear regressions for each group are shown. Bottom: the residual index plotted with length. The residual index is computed as the deviation of the weight datum from the overall regression line (pooling the two groups). +, Treatment; □, control.
A Whole Example
• Residual index: $F_{1,6} = 1.49$, $P<0.27$
  
  No significant treatment effect.

• ANCOVA: $F_{1,4} = 56.3$, $P<0.002$ (Test of interaction)

So, when the assumption of parallelism is not met, Huitema suggests computing and reporting separate regression slopes for all groups.
Reason #4

- Even if the assumptions of linear model hold for original variables, they will not hold for residuals. (independency and equal variance for the residuals will be violated)
Conclusions

• General linear models such as ANOVA, ANCOVA or simple linear regression involve the computation of various residuals are essential in order to verify the assumption of most statistical tests. However, there is no statistical reference to justify residual index as a correct alternative to the ANCOVA. And it is already shown by some scholars that ANOVA of residuals is incorrect procedure.
Suggestions

- A t-test or ANOVA of residuals should never be used in place of an ANCOVA to study condition or any other variables.

- The various design of ANCOVA or multiple regression of the original response variable should be generally used instead of using residuals as data.
Thank you!