

Lecture 31

Example: Consider $X \sim \text{Bin}(500, p)$ where we test $H_0 : p = 0.7$ versus $H_1 : p \neq 0.7$ at level $\alpha = 0.01$.

(a) Find the critical region of the test.

(b) Calculate the power at $p = 0.6$.

Example: In a two sample test of $H_0 : \mu_1 - \mu_2 = 3$ versus $H_1 : \mu_1 - \mu_2 > 3$, suppose that the data are normal, $m = n$ and $\sigma_1^2 = \sigma_2^2 = 84.0$. Can we choose m such that the test has level $\alpha = 0.01$ and $\beta = 0.05$ at $\mu_1 - \mu_2 = 5.0$? This question concerns *experimental design*.

In two sample problems, we can relax the normality assumption in the case of large samples.

Given X_1, \dots, X_m iid independent of Y_1, \dots, Y_n iid with m and n large (ie. $m, n \geq 30$), then the following statistic can be used for testing and the construction of confidence intervals.

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{s_1^2/m + s_2^2/n}} \sim \text{Normal}(0, 1)$$

where μ_1 and μ_2 are the respective means, and s_1 and s_2 are the respective sample std devs.

Example: A college interviewed 1296 students wrt summer incomes. Based on the results in the following table, test whether there is a difference in earnings between male and female students.

Students	n	\bar{X}	s
male	675	\$1884.52	\$1368.37
female	621	\$1360.39	\$1037.46

Example: The test scores of first year students admitted to college directly from high school historically exceed the test scores of first year students with working experience by 10%. A recent sample of 50 first year students admitted directly from high school has an average test score of 74.1% with std dev 3.8%. An indpt sample of 50 first year students with working experience yields an average test score of 66.5% with std dev 4.1%. Test whether a change has occurred.