Lecture 30

Errors in testing:

Decision	Parameter Space H_0 true	H_1 true
Reject H_0	Type I Error	
Do not reject H_0		Type II Error

Discussion questions:

- what is a good test?
- can we have a perfect test?

Example: We examine Type I error and Type II error in the earlier example where a defendent is accused of a crime in a court of law.

Probabilities associated with errors in testing:

Decision	**Parameter Space** H_0 **true**	H_1 **true**
Reject H_0	α	$1-\beta$
Do not reject H_0		β

Discussion points:

- α **is the** ***significance level*** **of a test**
- **we typically fix** α
- $1-\beta$ **is referred to as the** ***power*** **of a test**
- **we want the power to be large**
- α, β **are test properties; indpt of data**
- **note that in our examples,** H_0 **is** ***simple***
- **note that in our examples,** H_1 **is** ***composite***

Example: We return to the one sample problem where $X_1, \ldots, X_n$ are iid, $\sigma = 1.8$, $\alpha = 0.05$ and $n = 100$. We are interested in testing $\mathrm{H}_0 : \mu = 3$ versus $\mathrm{H}_1 : \mu > 3$.

(a) Find the *critical region* (rejection region).

(b) Calculate the power at $\mu = 3.2$.

(c) Calculate the power at $\mu = 3.5$.

(d) What happens in (b) when $n = 100 \to 400$?

I have mentioned previously that statistical practice relies heavily on computation. Here is a general simulation procedure that can be used to approximate power. Suppose that you have data $X_1, \ldots, X_n$ **and are testing** H_0 **versus** H_1**. Suppose that the critical region is** $\{\underline{X} : \; Q(\underline{X}) \geq a\}$ **and leads to an intractable power expression**

$$\begin{aligned} \text{Power} &= \mathrm{P}(\textbf{reject } \mathrm{H}_0 \mid \mathrm{H}_1 \textbf{ true}) \\ &= \mathrm{P}(Q(\underline{X}) \geq a \mid \mathrm{H}_1 \textbf{ true}) \end{aligned}$$

Step 0: set counter = 0

Step 1: generate $x_1, \ldots, x_n$ **under** H_1

Step 2: if $Q(\underline{x}) \geq a$**, increase the counter by 1**

Repeat Steps 1 and 2 M times and approximate

$$\text{Power} \approx \frac{\text{counter}}{M}$$