

Lecture 28

In this lecture, we examine five examples each of which does something different in the context of hypothesis testing.

To address the inference step (step 3 of hypothesis testing), we compute a *p-value* which is defined as the probability of observing a result as extreme or more extreme than what we observed given that H_0 is true (think about this!)

The convention is to reject H_0 and conclude H_1 if the p-value is less than 0.05. Sometimes a stronger level of evidence is required (e.g. 0.01).

Example: A shop sells coffee where the number of lb of coffee sold in a week is $\text{Normal}(320, 40^2)$. After advertising, 350 lb is sold in the following week. Has advertising improved business?

Example: A soup company makes soup in 10 oz cans. A sample of 48 cans has mean volume 9.82 oz and $s = 0.8$ oz. Can we conclude that the company is cheating? Test at level 0.01 significance.

Example: A coin is flipped 10 times and 8 heads appear. Is the coin fair?

Example: A coin is flipped 100 times and 60 heads appear. Is the coin fair?

Example: A paint is applied to tin panels and baked for one hour such that the mean index of hardness is 35.2. Suppose 20 panels are painted and baked for three hours, and their sample mean index of hardness is 37.2 with $s = 1.4$. Does baking for three hours strengthen panels? Assume normal data.

Single Sample Testing - Summary

Data	Test Statistic	Comments
normal, σ known	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0, 1)$	unrealistic
normal, σ unknown	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim t_{n-1}$	$N(0,1)$ when $n \geq 30$
non-specified, σ known, $n \geq 30$	$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0, 1)$	unrealistic based on CLT
non-specified, σ unknown, $n \geq 30$	$\frac{\bar{X}-\mu}{s/\sqrt{n}} \sim N(0, 1)$	based on CLT
Binomial	Binomial	
Binomial, $np \geq 5,$ $n(1-p) \geq 5$	$\frac{\hat{p}-p}{\sqrt{p(1-p)/n}} \sim N(0, 1)$	