Lecture 22

We now turn our attention to the expectation of functions of random variables.

Proposition: In the continuous case, using standard notation,

$$\mathrm{E}[g(X_1,\ldots,X_k)] = \int\cdots\int g(x_1,\ldots x_k)f(x_1,\ldots x_k)\ dx_1\cdots dx_k$$

In the discrete case, we replace the pdf f with the corresponding pmf p and we replace the multiple integral with a multiple sum.

Example: An instructor gives a quiz with two parts. For a randomly selected student, let X and Y be the scores obtained on the two parts respectively. The table gives the joint pmf $p(x, y)$ of X and Y:

p(x,y)	**y=0**	**y=5**	**y=10**	**y=15**
x=0	0.02	0.06	0.02	0.10
x=5	0.04	0.15	0.20	0.10
x=10	0.01	0.15	0.14	0.01

(a) What is the expected total score $\mathrm{E}(X+Y)$?

(b) What is the expected maximum score from the two parts?

(c) Are X and Y independent?

(d) Obtain $\mathrm{P}(Y = 10 \mid X \geq 5)$.

Example: We return to the discrete distribution described by the pmf $p(x, y, z)$

	X=1	**X=2**	**X=3**
Y=1	0.10	0.20	0.00
Y=2	0.00	0.05	0.05

$Z = 5$

	X=1	**X=2**	**X=3**
Y=1	0.00	0.30	0.10
Y=2	0.05	0.05	0.10

$Z = 6$

Obtain $\mathrm{E}(g)$ **where** $g(x, y, z) = xz$**.**

Problem: Annie and Alvie agree to meet for lunch between noon and 1pm. Denote Annie's arrival time by X and Alvie's by Y, and suppose X and Y are independent with pdfs $f_X(x) = 3x^2$ where $0 < x < 1$ and $f_Y(y) = 2y$ where $0 < y < 1$.

What is the expected time that the one who arrives first waits for the other person to arrive?

Recall that the conditional probability of event A given event B is given by

$$\mathrm{P}(A \mid B) = \mathrm{P}(AB)/\mathrm{P}(B).$$

Conditional probability is now extended to continuous rv's.

Definition: In the continuous case, using standard notation, the conditional density of X_1 given $X_2 = x_2, \ldots, X_k = x_k$ is given by

$$f_{X_1|X_2,\ldots,X_k}(x_1) = \frac{f_{X_1,\ldots,X_k}(x_1, \ldots, x_k)}{f_{X_2,\ldots,X_K}(x_2, \ldots, x_k)}$$

The definition can be extended in various ways including the discrete case.

Example: Recall the bivariate distribution on (X, Y) **given by the pdf** $f_{X,Y}(x, y) = 2(2x + 3y)/5$ **where** $0 < x, y < 1$**. Earlier we established the marginal density for** X **given by** $f_X(x) = 4x/5+3/5$ **where** $0 < x < 1$**. Suppose we observe** $X = 0.2$**. What is the conditional pdf of** Y**?**

Problem: The number of customers waiting for the gift-wrap service at department store is a rv X taking possible values 0, 1, 2, 3 and 4 with corresponding probabilities 0.10, 0.20, 0.30, 0.25 and 0.15. A random customer has 1, 2 or 3 packages for wrapping with probabilities 0.6, 0.3 and 0.1 respectively. Let Y be the total number of packages to be wrapped by customers waiting in line.

(a) Determine $\mathrm{P}(X = 3, Y = 3)$**.**

(b) Determine $\mathrm{P}(X = 4, Y = 11)$**.**