Lecture 20

Problem: The weight distribution of parcels is normal with mean value 12lb and std dev 3.5lb. The parcel service wants to establish a weight c beyond which there is a surcharge. What is the value of c such that 99% of parcels are at least 1lb under the surcharge weight?

Problem: The breakdown voltage of a randomly chosen diode is normally distributed with mean 40V and standard deviation 1.5V.

(a) What is the probability that the voltage of a single diode is between 39V and 42V?

(b) What value is such that only 15% of diodes have voltages exceeding that value?

(c) If four diodes are randomly selected, what is the probability that at least one has voltage exceeding 42V?

Definition: A rv X has a $\text{Gamma}(\alpha, \beta)$ distribution, $\alpha > 0$, $\beta > 0$, if it has pdf

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \qquad\qquad x > 0$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\ dx$

Discussion points:

- **pdf generally intractable**
- **contrast the range (**$x > 0$**) with the normal**
- **asymmetric**
- $\Gamma(\alpha)$ **is a constant**
- $\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1)$**,** $\Gamma(1) = 1$**,** $\Gamma(1/2) = \sqrt{\pi}$

Proposition: If $X \sim \mathrm{Gamma}(\alpha, \beta)$, then

- $\mathrm{E}(X) = \alpha\beta$
- $\mathrm{V}(X) = \alpha\beta^2$

The Exponential(λ) **distribution is a special case of the** Gamma(α, β) **where** $\alpha = 1$ **and** $\beta = 1/\lambda$**.**

Definition: A rv X **has an** Exponential(λ) **distribution,** $\lambda > 0$**, if it has pdf**

$$f(x) = \lambda e^{-\lambda x} \qquad x > 0$$

Discussion points:

- $\mathrm{E}(X) = \alpha\beta = 1(1/\lambda) = 1/\lambda$
- $\mathrm{V}(X) = \alpha\beta^2 = 1(1/\lambda)^2 = 1/\lambda^2$
- **the density is decreasing for** $x > 0$
- **the density is tractable; in particular the cdf** $F(x) = 1 - e^{-\lambda x}$ **for** $x > 0$

The Exponential distribution possesses a curious property known as the *memoryless* property. To appreciate the property, consider a rv X which is the lifespan of a lightbulb in hours where we assume that $X \sim \mathrm{Exponential}(\lambda)$. Then the probability that a used lightbulb (that has already lasted a hours) will last an additional b hours is given by

$$\mathrm{P}(X > a + b \ \mid X > a) =$$