

Lecture 18

Proposition: If $X \sim \text{Normal}(\mu, \sigma^2)$, then

$$Z = \frac{X - \mu}{\sigma} \sim \text{Normal}(0, 1)$$

A consequence is that any normal probability can be converted to a probability involving the standard normal. This means that we only need a single normal table instead of tables for all possible values of μ and σ .

Problem: The number of hours that people watch television is normally distributed with mean 6.0 hours and standard deviation 2.5 hours (first ask yourself if this is reasonable). What is the probability that a randomly selected person watches more than 8 hours of television per day?

Problem: The substrate concentration (mg/cm^3) of influent to a reactor is normally distributed with $\mu = 0.30$ and $\sigma = 0.06$.

- (a) What is the probability that the concentration exceeds 0.25?
- (b) What is the probability that the concentration is at most 0.10?
- (c) How would you characterize the largest 5% of all concentration values?

An amusing but real problem: My wife was expecting on June 1. My friends wanted me to go on a golf trip May 14, 15 and 16. What to do?

Proposition: Let $\eta(p)$ denote the $100p$ -th percentile of the standard normal distribution. Then the $100p$ -th percentile of the $\text{Normal}(\mu, \sigma^2)$ distribution is $\mu + \sigma\eta(p)$.

Example: Find the 25.78-th percentile of the $\text{Normal}(5, 100)$.

Proposition: Consider $X \sim \text{Bin}(n, p)$ where $np \geq 5$ and $n(1 - p) \geq 5$. Then we have the following approximation

$$X \sim \text{Normal}(np, np(1 - p))$$

Example: Obtain $P(X \geq 8)$ where $X \sim \text{Bin}(10, 1/2)$

- (a) exactly
- (b) using the normal approximation
- (c) using the normal approximation with a *continuity correction*.

Reminders:

(1) Probabilities associated with the $\text{Bin}(n, p)$ are sometimes difficult to evaluate. The following approximations are available:

(a) $\text{Poisson}(np)$ if n is large and p is small

(b) $\text{Normal}(np, np(1 - p))$ if $np \geq 5$ and $n(1 - p) \geq 5$

(2) Use a continuity correction whenever you need to approximate a discrete distribution with a continuous distribution.