Lecture 17

Problem: Consider the pdf of the rv Y

$$f(y) = \begin{cases} y/25 & 0 \le y < 5 \\ 2/5 - y/25 & 5 \le y < 10 \end{cases}$$

(a) obtain the cdf of Y

(b) calculate the 100p-th percentile of Y

(c) calculate $\mathrm{E}(Y)$

Problem: Let X be the time in hours that a reserved book is checked out by a randomly selected student. Suppose that X has the density function

$$f(x) = \begin{cases} x/2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) calculate $\mathrm{P}(X \leq 1)$

(b) calculate $\mathrm{P}(0.5 \leq X \leq 1.5)$

(c) calculate $\mathrm{P}(0.5 < X)$

Problem: A professor never finishes lectures before the end of the hour and always finishes within two minutes after the hour. Let X be the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) evaluate k

(b) what is the probability that the lecture ends within one minute of the end of the hour?

(c) what is the probability that the lecture continues beyond the hour for between 60 and 90 seconds?

(d) what is the probability that the lecture continues for at least 90 seconds beyond the end of the hour?

Problem: The cdf of checkout duration in minutes X is

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2/4 & 0 \le x \le 2 \end{cases}$$

(a) calculate $\mathrm{P}(0.5 \le X \le 1)$

(b) calculate the median of X

(c) calculate the pdf of X

(d) calculate $\mathrm{E}(X)$

Without doubt, the most important distribution in all of Statistics is the normal (Gaussian) distribution.

Definition: A rv X has a Normal(μ, σ^2) distribution if it has pdf

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

where $x \in \mathcal{R}$, $\mu \in \mathcal{R}$ and $\sigma > 0$.

Some talking points:

- **the normal is a family of distributions**
- **the density is symmetric about μ**
- **the density never touches zero**
- **the density is not tractable**
- **the parameters are interpretable: $\mathrm{E}(X) = \mu$ and $\mathrm{V}(X) = \sigma^2$**
- **data are often approximately normal**
- **the standard normal distribution is $\mathrm{Normal}(0, 1)$ and is typically represented by the rv Z**

To gain an understanding of the parameters μ and σ, sketch plots of the following distributions:

- $\text{Normal}(5, 1)$
- $\text{Normal}(7, 1)$
- $\text{Normal}(5, 10)$
- $\text{Normal}(5, 1/10)$

You must become familiar with the standard normal table (Table B.2 in the text). Calculate the following:

(a) $\mathrm{P}(Z \leq 3.02)$

(b) $\mathrm{P}(Z > 3.03)$

(c) $\mathrm{P}(Z < 3.025)$ **via interpolation**

(d) $\mathrm{P}(2.3 \leq Z \leq 2.6)$

(e) $\mathrm{P}(Z > -1)$

(f) z **such that 30.5% of** Z**-values exceed** z