Lecture 16

Review problem: A limousine can accommodate up to four passengers. The company accepts up to six reservations and passengers must have a reservation to travel. From records, 20% of passengers with reservations do not show.

(a) If six reservations are made, what is the probability that at least one passenger cannot be accommodated?

(b) If six reservations are made what is the expected number of available places when the limousine departs?

(c) Suppose that the pmf of the number of reservations R is

r	3	4	5	6
$p(r)$	0.1	0.2	0.3	0.4

Find the pmf of the number of passengers X who show up.

Definition: A rv is *continuous* if it takes on real values in an interval.

Example: Let X be the temperature in degrees Celsius at SFU.

Definition: Let X be a continuous rv. Then the *probability density function* (pdf) $f(x) \geq 0$ of X is such that

$$P(a \leq X \leq b) = \int_a^b f(x)\ dx \qquad \forall a < b$$

Proposition: The function $f(x)$ is a pdf if

1. $f(x) \geq 0$ **and**

2. $\int_{-\infty}^{\infty} f(x)\ dx = 1$

Problem: Verify that $f(x)$ is a pdf where

$$f(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x \leq 1 \\ 1/2 & 1 < x \leq 2 \\ 0 & 2 < x \end{cases}$$

Calculate $\text{Prob}(1 \leq X \leq 1.5)$**.**

Definition: A rv X has a Uniform(a, b) distribution if it has pdf

$$f(x) = \frac{1}{b-a} \qquad a < x < b$$

Special case: Uniform(0,1)

Definition: The *cumulative distribution function* (cdf) of a continuous rv X is given by

$$F(x) = \mathrm{P}(X \leq x) = \int_{-\infty}^{x} f(y)\ dy \qquad a < x < b$$

Definition: The 100p-th *percentile* of the continuous distribution with cdf $F(x)$ is the value $\eta(p)$ such that

$$p = F(\eta(p))$$

Definition: The *median* $\tilde{\mu}$ of the continuous distribution with cdf $F(x)$ is the 50-th percentile (i.e. $0.5 = F(\tilde{\mu})$).

Example: Find the median of the $\text{Uniform}(a, b)$ **distribution.**

Definition: The expected value of a continuous rv X with pdf $f(x)$ is

$$\mu = \mathrm{E}(X) = \int_{-\infty}^{\infty} x f(x)\ dx \qquad a < x < b$$

Proposition: If X is a continuous rv with pdf $f(x)$

$$\mathrm{E}(g(X)) = \int_{-\infty}^{\infty} g(x) f(x)\ dx$$

Definition: The variance of a continuous rv X with pdf $f(x)$ is

$$\mathrm{V}(X) = \mathrm{E}((X - \mathrm{E}(X))^2) = \int_{-\infty}^{\infty} (x - \mathrm{E}(X))^2 f(x)\ dx$$

Proposition: If X is a continuous rv, then as in the discrete case,

- $\mathrm{V}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2$
- $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$
- $\mathrm{V}(aX + b) = a^2\mathrm{V}(X)$