Lecture 15

Recall that the Binomial pmf may be difficult to calculate. It turns out the Binomial can sometimes be approximated by the Poisson.

Proposition: Without being rigorous, $\text{Bin}(n, \theta) \approx \text{Poisson}(n\theta)$ **if** n **is much larger than** $n\theta$**.**

Example: A rare type of blood occurs in a population with frequency 0.001. If n people are tested, what is the probability that at least two people have this rare blood type? Calculate the probability using the Binomial distribution and the Poisson approximation to the Binomial.

Example: A shipment of 5000 parts arrives where 0.5% of the parts are nonconforming. We randomly select 25 parts from the shipment and we reject the entire shipment if more than three of the selected parts are nonconforming. What is the probability that the shipment is accepted?

Recall that the Binomial distribution can be motivated by considering n independent trials where the probability of success on each trial is constant. Similarly, the Poisson distribution can be motivated by three assumptions which comprise the *Poisson process*. The assumptions of the Poisson process are these:

1. **events are indpt in non-overlapping intervals**

2. **events are *stationary***

3. **during small time intervals, the probability of a single event is proportional to the length of the time interval and the probability of more than one event is negligible**

Proposition: Let $p(x,t)$ be the probability of x successes in an interval of length t. Under the assumptions of the Poisson process

$$p(x,t) = \frac{(\lambda t)^x e^{-\lambda t}}{x!} \qquad x = 0, 1, \ldots$$

Example: A switchboard receives calls at a rate of three per minute during a busy period. Let X_t denote the number of calls in t minutes during a busy period. Assess whether the assumptions of a Poisson process are reasonable. Then, assuming the assumptions are reasonable, calculate the probability of receiving more than three calls in a two-minute interval during a busy period.