Lecture 14

Problem: A friend recently planned a camping trip. He had two flashlights, one that required a single 6-V battery and another that used two size-D batteries. He had previously packed two 6-V and four size-D batteries in his camper. Suppose that the probability than any particular battery works is p and that batteries work or fail independently of one another. Our friend wants to take just one flashlight. For what values of p should he take the 6-V flashlight?

Problem: A k-out-of-n system is one that functions if and only if at least k of the n individual components in the system function. If individual components function independently of one another, each with probability 0.9, what is the probability that a 3-out-of-5 system functions?

How many components do you expect to work in a 3-out-of-5 system?

Problem: Suppose that only 20% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions,

(a) at most six will come to a complete stop?

(b) exactly six will come to a complete stop?

(c) at least six will come to a complete stop?

(d) How many of the next 20 drivers do you expect to come to a complete stop?

Problem: A baseball player with a 300 average has 600 at-bats (attempts) in a season.

(a) Propose a pmf for the number of hits X.

(b) Is the probability distribution reasonable?

(c) What is the expected number of hits?

Definition: A rv X has a Poisson(λ) distribution, $\lambda > 0$, if it has pmf

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0, 1, 2, \ldots$$

The Poisson distribution is especially good at modelling rare events (more later).

For $X \sim \text{Poisson}(\lambda)$

- $\text{E}(X) = \lambda$
- $\text{V}(X) = \lambda$

Proof (first result):

$$
\begin{aligned}
\text{E}(X) &= \Sigma_{x=0}^{\infty}\ x\ \lambda^x e^{-\lambda}/x! \\
&= e^{-\lambda}\ \Sigma_{x=1}^{\infty}\ x\ \lambda^x/x! \\
&= e^{-\lambda}\lambda\ \Sigma_{x=1}^{\infty}\ \lambda^{x-1}/(x-1)! \\
&= e^{-\lambda}\lambda\ \Sigma_{y=0}^{\infty}\ \lambda^y/y! \\
&= e^{-\lambda}\lambda\ e^{\lambda} \\
&= \lambda
\end{aligned}
$$

Time for some fun (not related to Poisson)!

15 + 6

3 + 56

89 + 2

53 - 12

75 + 26

-7 - 9

123 + 5

Quick: Think about a __________ and a __________.