# Lecture 12

**Definition: The *cumulative distribution function* (cdf) of a random variable $X$ with probability measure $P_X$ is given by**

$$F_X(x) = P_X(X \leq x)$$

**Example: Consider three flips of a coin and let $X$ be the number of heads. Obtain the cdf of $X$.**

**Properties of a cdf $F$:**

**(1)** $F$ **is normed (i.e.** $F(-\infty) = 0$, $F(\infty) = 1$**)**

**(2)** $F$ **is monotone increasing**

**(3)** $F$ **is right continuous**

**Given a cdf corresponding to a discrete distribution, be able to determine the pmf.**

**Example:**

**Definition: The *expectation* of a discrete rv $X$ with pmf $p(x)$ is given by**

$$\mu \equiv \mathrm{E}(X) \equiv \sum_x x \ p(x)$$

**The expectation can be thought of the long run average of the random variable over hypothetical repetitions of the experiment.**

**Example: Consider the experiment consisting of three flips of a coin. Let $X \equiv$ the number of heads. Obtain $\mathrm{E}(X)$.**

**Example: Consider the experiment of tossing a die and let $X$ be the outcome. Obtain $\mathrm{E}(X)$.**

**Earlier it was stated that we view the expectation of a random $X$ as the long run average of $X$. Lets explore this statement by considering $N$ hypothetical repetitions of the experiment.**

**Proposition: The expectation of a function $g(X)$ corresponding to the discrete random variable $X$ with pmf $p(x)$ is given by**

$$\mathrm{E}(g(X)) = \sum_x g(x)\ p(x)$$

**Example: Consider the experiment of tossing a die and let $X$ be the outcome. Obtain $\mathrm{E}(X^2)$.**

**Proposition:** $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$

**Problem: A store orders copies of a weekly magazine for its magazine rack. Let $X$ be the weekly demand for the magazine with pmf**

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $p(x)$ | $\frac{1}{15}$ | $\frac{2}{15}$ | $\frac{3}{15}$ | $\frac{4}{15}$ | $\frac{3}{15}$ | $\frac{2}{15}$ |

**Suppose that the store owner pays \$1 for each copy of the magazine and the customer price is \$2. If leftover magazines at the end of the week have no salvage value, is it better for the owner to order three magazines or four magazines?**

**Is expectation always a reasonable criterion?**

**Problem for discussion: Suppose that you are given the chance to play a game a single time where the entrance fee is $1 million dollars. With probability 0.99, you lose and receive nothing. With probability 0.01, you win and receive $1 billion dollars. Should you play the game?**