Lecture 11

Coincidences are often misunderstood.

Example for discussion: Richard Baker left a shopping mall, found what he thought was his car and drove away. Later, he realized it was the wrong car and returned to the parking lot. The car belonged to another Mr Baker who had the same model of car, with an identical key! Police estimated the odds of this happening at one million to one.

- **Were the police correct?**
- **How astonished should we be?**

Example for discussion: Consider the case of twins who were separated at birth. They later meet as adults and are amazed that they share some striking characteristics (eg. they use the same toothpaste, their eldest children have the same names, they have the same job).

Should they be amazed?

Definition: A *random variable* (rv) is a function of the sample space.

Example: A coin is flipped three times. Let X be the number of heads.

Definition: A random variable is *discrete* if its outcomes are discrete.

Definition: A random variable that takes on the values 0 and 1 is *Bernoulli*.

Example: Consider the temperature in degrees Celsius. Let $Y = 1(0)$ if the temperature is freezing (not freezing).

Definition: The *probability mass function* (pmf) of a discrete random variable X is

$$p_X(X = x) = P(s \in S \ : X(s) = x)$$

Example: Consider the experiment consisting of three flips of a coin. Let $X \equiv$ the number of heads. Obtain the pmf for X.

Proposition: A pmf p_X satisfies

(1) $p_X(x) \geq 0$

(2) $\Sigma_x \, p_X(x) = 1$

Example: Let X be the sum of two dice. Obtain the pmf of X.

Example: Consider a batter with a 300 average. Let X be the number of at bats until the batter gets a hit. Obtain the pmf of X.

Problem: A library subscribes to two weekly magazines, each of which is suppose to arrive on Wednesdays. In actuality, the two magazines arrive independently with probabilities of arrival, $P(\text{Wed}) = 0.3$**,** $P(\text{Thu}) = 0.4$**,** $P(\text{Fri}) = 0.2$ **and** $P(\text{Sat}) = 0.1$**. Let** Y **be the number of days beyond Wednesday that it takes for both magazines to arrive. Obtain the pmf of** Y**.**

Problem: At the end of an exam, four textbooks are left behind. At the beginning of the next lecture, the four texts are randomly returned to the four students. Let X be the number of students who receive their own book. Obtain the pmf of X.