Lecture 09

Proposition: The number of *combinations* of r objects chosen from n distinct objects is

$$\binom{n}{r} = \frac{n^{(r)}}{r!} = \frac{n!}{r!(n-r)!}$$

Example: We can choose two of the symbols A, B, C, D and E in $\binom{5}{2} = \frac{5!}{2!(5-2)!} = 10$ **ways.**

Calculating combinations by hand: Try $\binom{30}{4}$.

Example: There are 20 people in a room. How many committees of four people can be chosen?

Anecdote regarding combination locks:

Proposition: There are $\binom{n}{r}$ ways of partitioning n distinct objects into a first group of size r and a second group of size $n - r$.

Corollary: $\binom{n}{r} = \binom{n}{n-r}$

Proposition: Let $n = n_1 + \cdots + n_k$. There are $\frac{n!}{n_1! n_2! \cdots n_k!}$ ways of partitioning n distinct objects into k distinct groups of sizes $n_1, n_2, \ldots, n_k$.

Example: How many ways can we partition the symbols A, B, C and D into distinct groups of sizes 1, 2 and 1?

Lets summarize: We have been developing counting rules, specifically $n!$, $n^{(r)}$, $\binom{n}{r}$ and $\frac{n!}{n_1!n_2!\cdots n_k!}$.

Whereas none of these rules are too difficult individually, the challenge is to use the counting rules to calculate probabilities.

Using the symmetry definition, recall that the probability of an event A is given by

$$P(A) = \frac{\textbf{number of outcomes where } A \textbf{ occurs}}{\textbf{total outcomes in the experiment}}$$

Problem: In a class of 100 students, 20 are female. If we randomly draw five students to form a committee, what is the probability that at least two of the committee members are female?

Problem: In a row of four seats, two couples randomly sit down. What is the probability that nobody sits beside their partner?

Problem: We roll a die. If we obtain a 6, we choose a ball from box A where three balls are white and two are black. If we do not obtain a 6, we choose a ball from box B where two balls are white and four are black.

(a) What is the probability of obtaining a white ball?

(b) If a white ball is chosen, what is the probability that it came from box A?

Problem: Five cards are dealt from a deck of 52 playing cards. What is the probability of

(a) three of a kind?

(b) two pair?

(c) straight flush?