Lecture 08

Independence: Lets begin thinking about independence in an informal way. Two events are independent if the occurrence or nonoccurrence of one event does not affect the probability of the other event.

Formally, and this is how you are required to prove independence, events A and B are independent if and only if

$$P(AB) = P(A)P(B)$$

Example: Suppose that I flip a coin and roll a die. What is the probability of obtaining a tail and a six?

Topic for discussion: Suppose that you go to a casino and you are watching roulette. You are thinking about placing a bet on either red or black. You have observed that the roulette wheel has resulted in a black number 6 times in a row. Do you bet red or black?

Does your opinion change if black comes up 100 times in a row?

More on independence: There is a connection between conditional probability and independence.

Proposition: Suppose $P(A) \neq 0$**,** $P(B) \neq 0$ **and** A **and** B **are independent. Then** $P(A \mid B) = P(A)$ **and** $P(B \mid A) = P(B)$**.**

The converse is also true.

Definition: Events $A_1, \ldots, A_k$ are *mutually independent* if and only if the probability of the intersection of any $2, 3, \ldots, k$ of these events equals the product of their respective probabilities.

Example: Consider the case of mutual independence of the events A_1, A_2, A_3 and A_4.

Example of pairwise independence but not mutual independence: Roll two dice and define

- $A_1 \equiv$ **first die is odd**
- $A_2 \equiv$ **second die is odd**
- $A_3 \equiv$ **sum of both dice is odd**

The birthday problem: Amongst 30 people, what is the probability that at least two of them share a common birthday?

Generalize the problem to n people.

Basic combinatorial results:

Proposition: The number of *permutations* of n distinct objects is $n! = n(n-1)(n-2)\cdots 1$

Example: We can permute symbols A, B and C in $3! = 6$ ways.

Definition: $0! = 1$**.**

Proposition: The number of permutations of r objects chosen from n distinct objects is $n^{(r)} = n!/(n-r)!$

Example: We can permute two of the symbols A, B, C, D and E in $5^{(2)} = 5!/(5-2)! = 120/6 = 20$ ways.