

# Lecture 07

Symmetry definition of probability:

In the case of a finite number of equally likely outcomes in an experiment,

$$P(A) = \frac{\text{number of outcomes leading to } A}{\text{number of outcomes in the experiment}}$$

Example: Roll two dice. Let  $A$  be the event that the sum is 10.

Discussion points:

Frequency definition of probability:

In hypothetical identical trials of an experiment,

$P(A)$  = the long term relative frequency of  $A$

Example: Roll two dice  $n$  times. Let  $A$  be the event that the sum is 10.

Discussion points:

**Problem:** If 85% of Canadians like either baseball or hockey, 63% like hockey and 52% like baseball, what is the probability that a randomly chosen Canadian likes both hockey and baseball?

**Conditional probability (an important topic):**

**The conditional probability of  $A$  given  $B$  is**

$$P(A | B) = \frac{P(AB)}{P(B)}$$

**provided that  $P(B) \neq 0$ .**

**Problem: Suppose that I roll a die and tell you that the result is even. What is the probability that the outcome is a 6?**

**Problem:** The probability of surviving a transplant operation is 0.55. If a patient survives the operation, the probability that the body rejects the transplant within a month is 0.2. What is the probability of surviving both critical stages?

Confusion of the inverse:  $P(A | B) \neq P(B | A)$

A patient has a lump in her breast. A physician believes that there is a 1% chance that the lump is malignant. A mammogram is positive where mammograms are accurate 80% of the time when lumps are malignant and mammograms are accurate 90% of the time when lumps are benign. The test comes back positive.

What is your opinion concerning the probability of the malignancy of the lump?

**Problem:** In each box of my favourite cereal, there is a prize. Suppose that the cereal company distributes 10 different prizes randomly in the boxes of cereal. If I purchase five boxes of cereal, what is the probability that I obtain five different prizes?

The Monty Hall problem: On the game show “Let Make a Deal”, a contestant is given the choice of three doors. Behind one door is a grand prize (e.g. a car) and behind the other two doors are gag gifts. The contestant picks a door, and Monty (who knows what is behind all of the doors), reveals a gag gift by opening one of the two doors that the contestant has not chosen. Monty then gives the contestant the choice of switching doors between the remaining two unopened doors. Should the contestant switch?